

## B++ SPLINES AND ISOGEOMETRIC ANALYSIS

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Isogeometric Analysis (IGA) based on NURBS has been originally introduced and developed by T.J.R. Hughes et al.[1] in 2005. In this paper, B++ splines (Boundary Plus Plus Splines) and isogeometric analysis using B++ splines for trimmed surfaces and solid CAD models is proposed. We show that any trimmed NURBS surface (or trimmed solid CAD model) with complex topologies can be converted into B++ splines that incorporate the control points of the trimmed curves (or specific points at the boundaries of the trimmed solid model) and a set of enriched control points. Each B++ spline basis function is a linear combination of standard NURBS basis functions. The displacement function shares the same B++ spline basis function. Thus, nonconforming control variables that are related with enriched control points are introduced in order to enrich the approximated space. The present method is an enhanced immersed boundary method[2], which stays within the standard Ritz-Galerkin framework, fulfills Dirichlet boundary conditions and is easy assemble for multiple patches. Emphasis is placed on the construction of an invertible transformation matrix that related the control points of a trimmed NURBS surface (or a trimmed solid CAD model) to those of the trimming curves at the boundaries (or specific boundary points on the solid model). In doing so, the resulting B++ spline basis functions associated with the control variables of the trimming curves interpolate the Dirichlet boundary, while the control variables of the enriched basis functions vanish at the boundaries. Fig.1 depicts a B++ spline basis function corresponding to a boundary control point of a trimmed NURBS surface, which is conforming to the boundaries of the surface. Fig.2 shows an enriched B++ basis function that vanishes at the boundaries. With examples of complex topologies that could be described by employing B++ splines, the effectiveness and robustness of the proposed method are demonstrated.

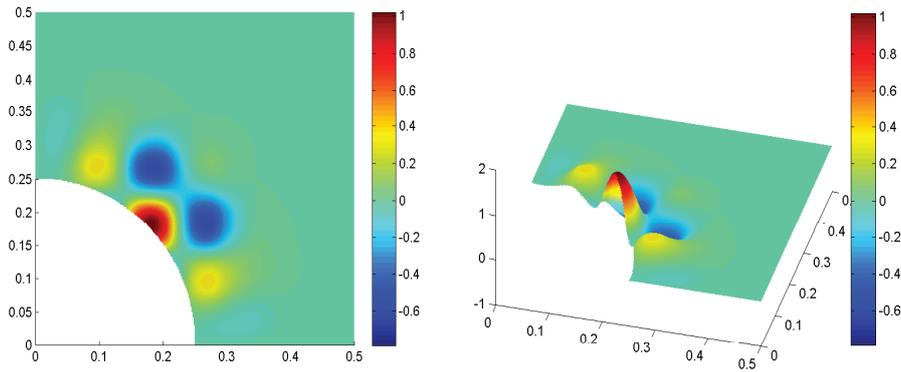


Figure 1: A B++ spline basis function corresponding to a boundary control point of a trimmed NURBS surface, which is conforming to the boundaries of the surface.

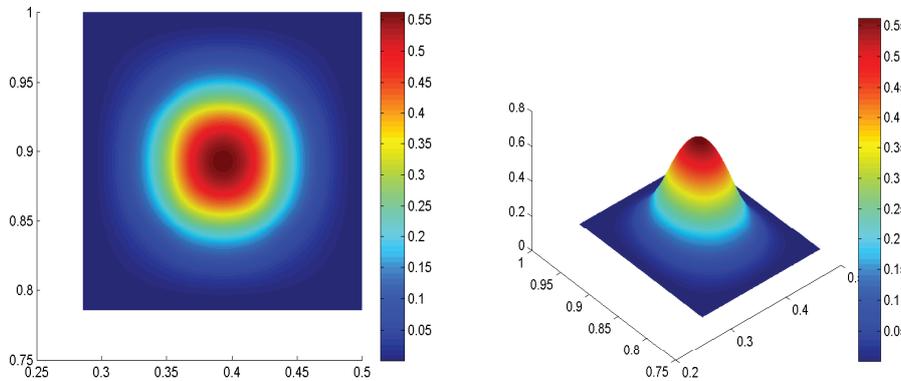


Figure 2: An enriched B++ basis function that vanishes at the boundaries.

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