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SHARP INTERFACE APPROACH IN TOPOLOGY OPTIMIZATION OF CONTACT PROBLEMS

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The paper deals with the shape and/or topology optimization for an elastic body in unilateral contact with a rigid foundation. The contact phenomenon with Tresca friction between the surfaces of the elastic or rigid bodies is described by the elliptic variational inequality. This optimization problem consists in finding such topology of the domain occupied by the body and/or the shape of its boundary that the normal contact stress along the boundary of the body is minimized. The volume of the body is bounded.

In structural optimization the level set method [1,4,6,8] used to be employed in numerical algorithms for tracking the evolution of the domain boundary on a fixed mesh and finding an optimal domain. This method is based on an implicit representation of the boundaries of the optimized structure, i.e., the position of the boundary of the body is described as an isocountour of a scalar function of a higher dimensionality. In standard level set approach the evolution of the domain boundary is governed by Hamilton - Jacobi equation. The speed vector field driving the propagation of the level set function is given by the Eulerian derivative of the cost functional with respect to the variations of the free boundary. Applications of the level set methods in structural optimization can be found, among others, in [6,8]. To improve the efficiency of the standard level set method different approaches are considered in literature, among others phase field methods [2,3,5,7,9,10,11,12,13].

In the paper phase field approach combined with level set method is proposed to regularize topology optimization problem for unilateral elastic contact system and to solve it numerically rather than standard level set method. Material density function is a variable subject to optimization. This approach consists in using Ginzburg Landau free energy term [2,3,9,10,12] as the regularization term rather than the perimeter constraint term. Although the proposed regularization for topology optimization of contact problems is more complicated than the perimeter one it has advantages comparing to the standard one. It leads to optimal topologies having suitable smoothness [3,11,13]. The derivative formula of the cost functional with respect to the material density function is calculated and is employed to formulate a necessary optimality condition for the topology optimization problem. Modified reaction-diffusion equation updating the level set function is derived. Moreover the cost functional derivative is employed to calculate a descent direction in the numerical algorithm. Details of numerical implementation are provided. Numerical examples are provided and discussed.

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