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A POSTERIORI ERROR CONTROL FOR BEM IN 2D-ACOUSTICS

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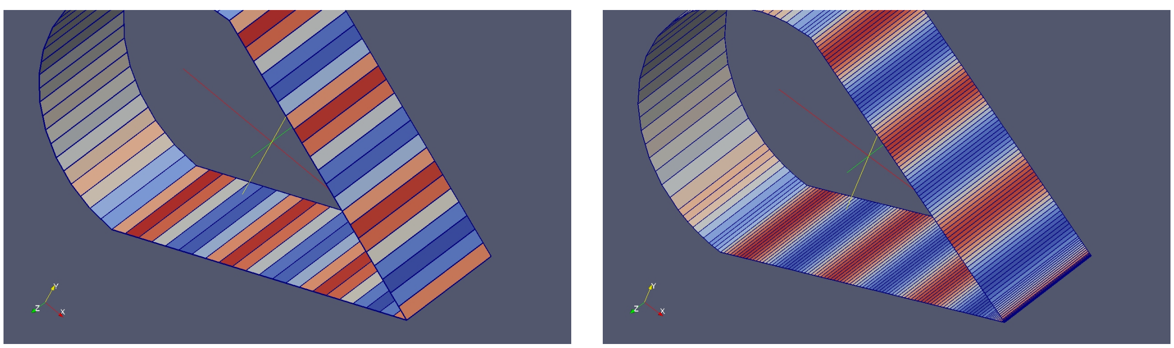
The main advantage of Boundary Element Method (BEM), based on boundary integral equation formulations, is to reduce the discretization to the boundary of the domain and to exactly take into account the radiation conditions to infinity. Hence, BEMs are well suited to deal with wave propagation problems. However, standard BEMs lead to fully-populated system matrices thus limiting the capabilities of the method to deal with real-life applications. To overcome those limitations, recent works on the acceleration of BEM with the Fast Multipole Method have been done. The FMM permits to reduce the CPU time and memory requirements drastically, and therefore to deal with larger problems. So far, most of the efforts have been dedicated to the acceleration of the solution of the BEM. In the context of oscillatory kernels the BEM lacks of automatic tools to guarantee the accuracy of the computed solution and more specifically of an efficient and reliable *a posteriori* error estimate. The aim of this work is to select, analyze and test different kind of *a posteriori* error estimate tools for acoustics in the 2-D case.

We consider in this work the Helmholtz equation $\Delta U + k_{\mathrm{wave}}^2 U = 0$ for both exterior Dirichlet and Neumann boundary conditions on the scattering object (k_{wave} is the (real) wave number). We decompose the field U as the sum of the incoming wave u_i and the scattered wave u (satisfying the Sommerfeld radiation condition). The problem is formulated with an integral equation of the first kind (i.e. the Dirichlet problem is solved with a Single Layer Potential and the Neumann problem with the normal derivative of the associated Double Layer Potential). Our main goal is to propose an estimate η which is equivalent to the error $e = \|u - u_{\mathrm{h}}\|$ where $\|.\|$ is the norm associated to the space where the solution u lives ($\mathrm{H}^{-1/2}(\Gamma)$ for the Dirichlet Problem, $\mathrm{H}^{1/2}(\Gamma)$ for the Neumann problem).

Our methodology is an extension to the acoustics problems of the averaging *a posteriori* estimate proposed by D. Praetorius et al. ([1] and [2]). In a first step, we consider a spatial reconstruction. The idea is to compute the solution u_h on the mesh $\mathcal{T}_\mathrm{h}$ which is twice finer than the initial one $\mathcal{T}_\mathrm{H}$. This solution is computed on a $\mathrm{P}^k(\mathcal{T}_\mathrm{h})$ approximation space. Then, using an appropriate projection operator $\mathrm{P}_\mathrm{H} : u_\mathrm{h} \mapsto \mathrm{P}_\mathrm{H} u_\mathrm{h}$, the solution on a $\mathrm{P}^{k'}(\mathcal{T}_\mathrm{H})$ (with $k' > k$) approximation space of the coarse mesh is reconstructed. In our case we consider $k = 0$ (resp. $k = 1$) and $k' = 1$ (resp. $k' = 2$) for the Dirichlet (resp. Neumann) problem. Under some regularity assumptions and some conditions on the reconstruction space, there exists $(C_1, C_2) > 0$ such that $C_1 \|u_\mathrm{h} - \mathrm{P}_\mathrm{H} u_\mathrm{h}\| < e < C_2 \|u_\mathrm{h} - \mathrm{P}_\mathrm{H} u_\mathrm{h}\|$. The difficulty is that the norm of the space is either not local, or simply cannot be computed. As a result, this error estimate cannot be used in a fully automatic adaptative mesh refinement strategy. To overcome this problem, for example for the Dirichlet problem, we use an equivalent weighted L^2-norm: if K is an boundary element, we define the localized error estimate η as $\eta^2 = \sum_K \|H^{\frac{1}{2}}(u_\mathrm{h} - \mathrm{P}_\mathrm{H} u_\mathrm{h})\|^2_{\mathrm{L}^2(K)}$ (where H is the length of the element K). This error estimate is used in conjunction with a fully automatic adaptative mesh refinement strategy. We consider two strategies: (i) to refine the cells where the error estimate is greater than X% of the maximum value of all the computed error estimates (see Fig. 1, X = 90) or (ii) the Dörfler marking.



(a) **(b)**

Figure 1: Solution of the Dirichlet problem for an incoming plane wave (coming from the right): (**a**) initial mesh; (**b**) mesh after 10 steps of the adaptative mesh refinement strategy.

Finally, the efficiency of this fully automatic adaptative mesh refinement strategy is shown on different kind of scattering objects like a disc or a Helmholtz resonator. We observe that the adaptative algorithm restores the optimum convergence rate, including for singular geometries. We also observe for problems with known analytic solutions that the error estimates are close to the real errors (i.e. the efficiency constants are close to 1).

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