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COMPATIBLE DISCRETE OPERATOR SCHEMES FOR STOKES PROBLEM ON POLYHEDRAL MESHES

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Compatible Discrete Operator (CDO) schemes belong to the class of compatible (or mimetic, or structure-preserving) schemes. Their aim is to preserve structural properties of the underlying PDE at the discrete level. Degrees of freedom are defined using De Rham maps depending on the physical nature of the field to discretize, and a distinction is operated between topological laws (that are discretized exactly) and constitutive relations (that are approximated). CDO schemes are formulated using discrete differential operators for the topological laws and discrete Hodge operators for the constitutive relations. The discrete Hodge operator is the key operator in the CDO framework. Contrary to discrete differential operators, the design of discrete Hodge operators is not unique. Each design leads to a different scheme. CDO schemes involve two meshes: a primal mesh (which is the only one seen by the end-user) and a dual mesh. The discrete Hodge operator links quantities defined on the primal mesh to quantities defined on the dual mesh.

CDO schemes have been recently analyzed in [1] for elliptic problems on polyhedral meshes. Design properties of the discrete Hodge operator have been identified, both at the discrete level and when using reconstruction functions. The design properties, which essentially hinge on stability and $\mathbb{P}_0$-consistency, lead, in particular, to first-order convergence in energy and complementary energy norms for smooth solutions. Second-order L^2-norm error estimates have also been derived for the potential.

We first present the CDO framework. Then, we derive two CDO schemes for steady, incompressible Stokes flows. The starting point of both schemes is the rotational form of the viscous term in the momentum balance. The (constant) density and the (constant) viscosity each lead to a corresponding discrete Hodge operator. In the first scheme, the degrees of freedom for the pressure are positioned at the vertices of the primal mesh, and the velocity at the primal edges (thereby viewing the velocity as a circulation). This scheme is called vertex-based. In the second scheme, the degrees of freedom for the pressure are positioned at the primal cells (or equivalently the dual vertices), and the velocity at the primal faces (thereby viewing the velocity as a mass flux). This scheme is called cell-based, and is closely related to the recent work of Kreeft and Gerritsma [3]. We present the main theoretical results for the CDO schemes, including discrete stability, well-posedness, and error bounds leading to first-order convergence rates for smooth solutions. Then, we present numerical results on various polyhedral meshes. Reconstruction functions introduced by Codecasa *et al.* [2] are used to build the discrete Hodge operators.

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