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VERIFICATION OF SHELL FINITE ELEMENTS IN THE GIRKMANN BENCHMARK PROBLEM

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Shell finite element formulations involve various explicit and implicit modelling assumptions that extend beyond the limits of mathematical convergence theory currently available, see [1]. However, thanks to modern computation technology, such as the *hp*-adaptive finite element method [2, 3], shell analysis can also be based directly to three-dimensional elasticity theory. Such approach rules out the modelling errors arising from the simplifications of dimensionally reduced structural models but requires in general more degrees of freedom for the discrete model. Also, if simplified representations of the stress state such as the stress resultants are needed, they must be post-processed from the three-dimensional stress field.

A model problem called the *Girkmann problem*, which was revived some time ago, highlights the above complications rather dramatically, see [4, 5]. The problem involves a structure consisting of a spherical dome stiffened by a foot ring under a dead gravity load. The task is to determine the values of the shear force and the bending moment at the junction between the dome and the ring as well as the maximum bending moment in the dome.

The problem was initially presented and solved analytically in a text book by Karl Girkmann. More recently, in the bulletin of the International Association of Computational Mechanics (IACM), the problem was posed as a computational challenge to the finite element community. The purpose of the challenge was to find out how the process of verification, that is the process of building confidence that an approximative result is within a given tolerance of the exact solution to the mathematical model, is carried out by the community given the Girkmann problem. The results, that are summarized in [4], without attribution and details on how verification was actually performed, are scandalous. Out of the 15 results submitted, 11 have a very large dispersion and are not within any acceptable tolerance of the reference values computed in [4, 5] using different models and formulations.

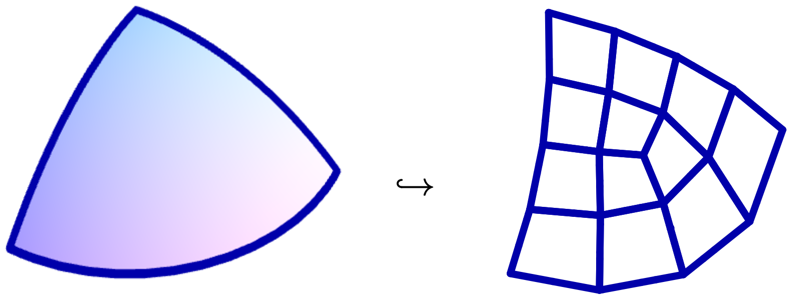

Figure 1: Bilinear finite element representation of the Girkmann dome.

So far detailed verification studies have been published for the axisymmetric models based on elasticity theory as well as axisymmetric dimensionally reduced models. In [4], the *p*-version of FEM was used in conjunction with the extraction of stress resultants to compute accurate values for the quantities of interest. Similar approach with the *hp*-version of FEM was taken in [5], where also the axisymmetric *h*-version with selective reduced integration was successfully employed to discretize the dimensionally reduced model.

In the present work, we use the Girkmann problem to benchmark general 3D shell elements. Namely, we model a quarter of the dome as shown in Fig. 1 and use different type of shell elements to approximate the deformation. We use the bilinear and biquadratic shell elements of LUSAS finite element analysis software as well as the bilinear shell element proposed in [6].

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