11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

# TRIANGULAR-FAN-BASED ALGORITHM FOR COMPUTING THE CLOSURE CONDITIONS OF PLANAR LINKAGES

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**Key words:** *Position analysis, bilateration, rigidity, isomorphism, kinematic*

The position analysis of a planar mechanism is based on obtaining the roots of its *characteristic polynomial*. In general, this polynomial is the result of a system of kinematic equations which they are derived from closure condition of the mechanism, widely known as independent kinematic loop equations or loop closure equations . This way of solving the position analysis of kinematic chains introduces complex variable eliminations, and in general trigonometric substitutions. Recently, the use of methods based on bilateration to solve the position analysis, has been shown to avoid these variable eliminations and trigonometric substitutions in planar mechanism. In this work[1] it is shown how this method based on bilateration can be use to automatically generate closure conditions of a planar mechanism.

Introduccing bilateration problem (see [1]) in the method it is possible in a systematic way, to get a set of bilateration equations treating the mechanism as a graph, by a sequence of eliminations of a common vertex of a set of adjacent triangles which forms a subgraph that is called *triangles fan*.

Let us denote a point as $P_i$, $\overline{P_iP_j}$ the edge defined by $P_i$ and $P_j$, and $\Delta_{i,j,k}$ the triangle defined by $P_i$, $P_j$, and $P_k$. The bilateration problem consists in finding the feasible locations of a point, say $P_k$ , given its distances to two other points, say $P_i$ and $P_j$ , whose locations are known. As a consequence, the unknown squared distance between $P_j$ and another point $P_l$ can be obtained with a concatenation of two bilaterations (See [3] for more details).

A planar *graph* $G = (V, E)$ consist of a set $V$ of nodes(vertices or points) and a set $E$ of edges, where $|V| = v$ and $|E| = e$. Define a *configuration* $P : V \to \mathbb{R}^n$ to be

[1]This work has been done under project IPRES, DPI2011-29660-C04-01 of the Spanish National Research Program, with partial FEDER funds.

an assignment of the vertices $V$ to points in $\mathbb{R}^n$. This work will be focused on $\mathbb{R}^2$. A *framework* $F = (G, P)$ refers to a *graph* $G$ with a *configuration* $P$. Let us denote a bitriangle $T_{i,j,k,l}$ composed by two triangles $\Delta_{i,j,k}$ and $\Delta_{i,k,l}$ sharing one edge. If we consider bitriangle $T_{i,j,k,l}$ as a rigid graph $S$, keeping the Laman condition(see [2]) by inserting the edge $\overline{P_jP_l}$, we can delete the *common point* $P_i$ or $P_k$ to leave a rigid equivalent graph. The distance of the new generated edge $\overline{P_jP_l}$ is obtained from concatenation of two bilaterations (See [3]).

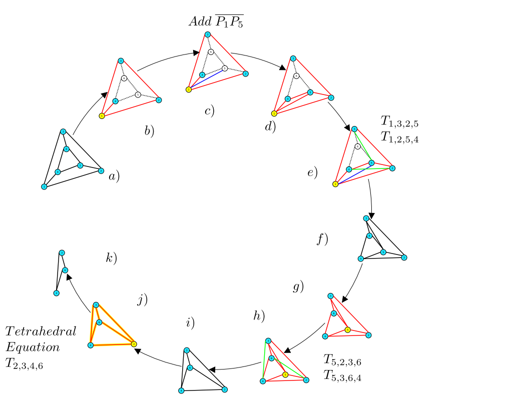


**Figure 1**: Example 5-Link Baranov Truss

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Let us denote a $\Xi_{i,j,k,l,\ldots,N+2}$ *triangle fan* as a set of connected bitriangles that share one central point(*common point*) which its N triangles has N+2 points. If we consider the *triangle fan* $\Xi_{i,j,k,l,\ldots,N+2}$ as a rigid graph $S$, keeping the Laman condition by inserting in every bitriangle $T_{i,j,k,l}$ the edge $\overline{P_jP_l}$ such that the generated edge do not create a cycle with the other generated edges and existing edges $\overline{P_jP_l}$ in the graph $S$, we can delete the *common point* to leave a rigidly equivalent graph without changing any essential rigid properties of initial graph. The recognition of a *triangles fan* $\Xi_{i,j,k,l,\ldots,N+2}$ in a graph $G$ is treated as a subgraph isomorphism problem where it determines whether graph $G_1$ is a subgraph of $G_2$.

The way to find a *triangles fan* $\Xi_{i,j,k,l,\ldots,N+2}$ in the graph $G$ is to select the point with all incident points to get a subgraph $S$, get all cycles(closed trail of vertices) of three vertices and check if all share at least one edge with any other one. The algorithm uses the *triangles fan* $\Xi_{i,j,k,l,\ldots,N+2}$ as a subgraph $S$ to eliminate points and add edges which keep the general graph $G$ over-rigid. First it finds the point $P_i$(*common point*) with the maximum incidence from the graph $G$, get the subgraph $S$ (*triangles fan* $\Xi_{i,j,k,l,\ldots,N+2}$) of that point, generate the edges of every bitriangle of the subgraph generating their closure conditions and remove the *common point* in the graph. The algorithm uses this operation until the remaining graph is a single triangle (See Fig. 1). The result is a set of equations which by sequential substitution we can get the closure conditions.

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