

APPROXIMATING COUPLER CURVES USING STRIP TREES

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For the mechanisms considered under the title linkages, coupler curve is the path traced by one of the point on the coupler link considered as an output of the mechanism which is joined to a fixed link. The equation of the coupler curve generated can be obtained solving a set of equations which describes distance constancy between all points of a mechanism and this coupler curve is the eliminant of these equations. The proposal to this work¹ is to approximate coupler curves using strip trees.

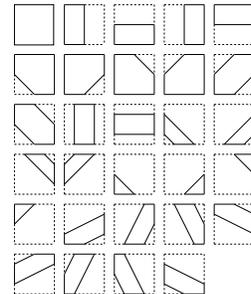


Figure 1: Bounding polygons

Strip trees were introduced by Ballard in the early eighties as a multiresolution data structure for representing curves [1]. The main concept in strip trees is to represent each piece of the curve by a bounding quadrilaterals that contains the piece. This representation is done in a hierarchical fashion starting from the whole curve so we get a tree of oriented quadrilateral, each quadrilateral containing a piece of the curve.

The multi-resolution representation provided by a strip tree can be used to solve several practical problems efficiently, including displaying a curve at a given resolution, intersecting two curves, computing the length of a curve at a given resolution, testing the proximity of a point to a curve, and testing whether a point is inside a region bounded by a curve.

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The idea is to obtain the coupler curves from the closure condition. The closure condition of planar linkages with mobility 1 can be compactly obtained using Distance Geometry (see [2]). This has been shown to be a valuable alternative to formulations based on sets of independent loop equations. These closure conditions are compact scalar equations (whose number is equal to the coupling number of the analyzed linkage typically 1) involving squared distances and signs of oriented areas combined through additions products and squared roots. Squared roots play a fundamental role as they are responsible for the compactness of the obtained closure expressions. These closure conditions can be seen as a parametric representation of the configuration space of the linkage.

In this work, we show how to compute a strip tree representation for general coupler curves, using affine arithmetic (see [3][4]) to find a good bounding geometry. Instead of bounding quadrilateral, the method uses a bounding polygon (See Fig. 1) which is the result of the intersection between bounding parallelepiped and the horizontal plane (See Fig. 2). As example Fig. 3 shows parallelepipeds covering a curve computed with our implementation.

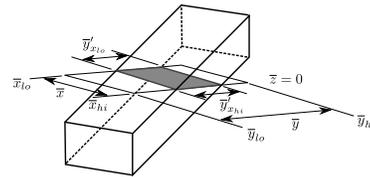


Figure 2: the intersection between bounding parallelepiped and the horizontal plane

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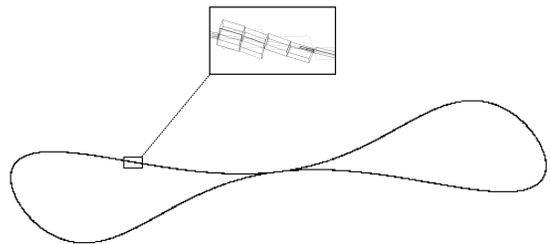


Figure 3: Parametric curve