11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

A MULTICRITERIA METHOD FOR TRUSS OPTIMIZATION

Ngọc Trình Trần*[1], Manfred Staat[1], Georgios E. Stavroulakis[2]

[1]Aachen University of Applied Sciences, Biomechanics Laboratory,
Institute of Bioengineering, Heinrich-Mußmann-Straße 1, D-52428, Jülich, Germany
{trinh.tran, m.staat}@fh-aachen.de
www.fh-aachen.de/forschung/institut-für-bioengineering/
[2]Technical University of Braunschweig, Institute of Applied Mechanics,
Bienroder Weg 87, Gebäude 1411, Campus Nord, D-38106 Braunschweig, Germany
and Technical University of Crete, GR-73100 Chania, Greece

Key Words: *Sizing optimization, Limit and shakedown analysis*

Introduction: We propose a multicriteria approach to the optimization of trusses. Stress and cross sectional areas of bars are considered as independent variables. This is a distinguishing point of this method compared to [1,2]. Using two independent variables makes the equilibrium equations become nonlinear. The problem is solved by the Optimization Toolbox of Matlab. In this paper, we will consider the nine-bar truss of [3] as an example.

Minimum volume of trusses at elastic state

The minimum strain energy combined with the equilibrium equations provide enough equations to determine the stress in either statically determinate or indeterminate trusses. Therefore the truss optimization problem at the elastic state can be written as follows:

$$\min \sum_{i=1}^{n} t_i l_i + \min \sum_{i=1}^{n} \frac{1}{2E} s_i^2 t_i l_i \tag{1}$$

s.t.

$$\sum\nolimits_{i=1}^{n_{ij}} s_i t_i \cos\alpha_{ij} - f_{jx} = 0 , \quad j = 1,2,...,n_{jx} \tag{1a}$$

$$\sum\nolimits_{i=1}^{n_{ij}} s_i t_i \sin\alpha_{ij} - f_{jy} = 0 , \quad j = 1,2,...,n_{jy} \tag{1b}$$

$$-s_a \le s_i \le s_a \tag{1c}$$

$$0 \le t_i \tag{1d}$$

where: t_i, l_i, s_i are the cross sectional area, length and stress of $i-th$ bar, respectively ,α_{ij} is the angle between $i-th$ bar connected to $j-th$ node with x-axis; f_{jx}, f_{jy} are applied loads acting on $j-th$ node; E is the Young's modulus, s_a is the allowable stress of material.
The above problem (1) is considered as a multicriteria optimization problem [5].

Minimum volume of trusses at plastic limit state

In this case, the limit load is the quantity which we need to determinate. Therefore, instead of using the condition of $\min V$, we use another weak condition in constraints: volume of material of trusses is less than a given source of material V_0. The problem is formulated as:

$$\max \mu \left[\sum_{j=1}^{n_{jx}} f_{jx} + \sum_{j=1}^{n_{jy}} f_{jy} \right] + \min \sum_{i=1}^{n} \frac{1}{2E} s_i^2 t_i l_i \qquad (2)$$

s.t.

$$\sum\nolimits_{i=1}^{n_{ij}} s_i t_i \cos \alpha_{ij} - \mu f_{jx} = 0 \,, \quad j = 1,2,...,n_{jx} \qquad (2a)$$

$$\sum\nolimits_{i=1}^{n_{ij}} s_i t_i \sin \alpha_{ij} - \mu f_{jy} = 0 \,, \quad j = 1,2,...,n_{jy} \qquad (2b)$$

$$-s_0 \le s_i \le s_0 \qquad (2c)$$

$$0 \le t_i \qquad (2d)$$

$$\sum\nolimits_{i=1}^{n} t_i l_i \le V_0 \qquad (2e)$$

where μ is the load factor.

Minimum volume of trusses at shakedown state

In case of a truss subjected to a load domain with NV vertices, the load vector can be described as a linear combination, and the problem is formulated as:

$$\max \sum_{L=1}^{NV} \mu \left[\sum_{jx=1}^{n_{jx}} f_{jx} + \sum_{jy=1}^{n_{jy}} f_{jy} \right]_L + \min \sum_{i=1}^{n} \frac{1}{2E} s_i^2 t_i l_i \qquad (3)$$

s.t.

$$\sum\nolimits_{i=1}^{n_{ij}} s_i t_i \cos \alpha_{ij} - \mu \left(f_{jx} \right)_L = 0, \quad j = 1,2,...,n_{jx}, \quad L = 1,2,...,NV \qquad (3a)$$

$$\sum\nolimits_{i=1}^{n_{ij}} s_i t_i \sin \alpha_{ij} - \mu \left(f_{jy} \right)_L = 0, \quad j = 1,2,...,n_{jy}, \quad L = 1,2,...,NV \qquad (3b)$$

$$\sum\nolimits_{i=1}^{n_{ij}} s_i^r t_i \cos \alpha_{ij} = 0 \,, \qquad \sum\nolimits_{i=1}^{n_{ij}} s_i^r t_i \sin \alpha_{ij} = 0 \qquad (3c)$$

$$s_{i,\max} + s_i^r \le s_0 \qquad (3d)$$

$$-s_{i,\min} - s_i^r \le s_0 \qquad (3e)$$

$$0 \le t_i \qquad (3g)$$

$$\sum\nolimits_{i=1}^{n} t_i l_i \le V_0 \qquad (3h)$$

where s_i^r is the residual stress; s_i is the elastic stress in the *i-th* bar correspondent with type of β*-th* loading, $s_{i,\max} = \max\left(s_i\right)_L$, $s_{i,\min} = \min\left(s_i\right)_L$.

References:

[1] P.W. Christensen, A. Klarbring: *An Introduction to Structural Optimization*. Springer (2008).

[2] M.P. Bendsøe, O. Sigmund: *Topology Optimization – Theory, Methods and Applications*. Springer, Berlin (2004).

[3] S. Kaliszky, J. Lógó: Optimal plastic limit and shakedown design of bar structures with constrains on plastic deformation. *Engineering Structures*, Vol. **19** (1), 19-27, 1997.

[4] S. Boyd, L. Vandenberghe: *Convex Optimization.* Cambridge University Press, Cambridge (2004).