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INTERIOR PENALTY FINITE ELEMENT METHODS FOR HIGH-ORDER LOCAL BOUNDARY CONDITIONS

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Local boundary conditions are used to mimick the solution in presence of an infinite exterior in diffusion problems or time-harmonic scattering problems, in highly conducting bodies or thin layers. We consider symmetric local boundary conditions in $\mathbb{R}^2$, which take the following form of Dirichlet-to-Neumann maps [4, Eq. 3.14]

$$\partial_n u(x) + \sum_{j=0}^{J} (-1)^j a_j \, (\partial_\tau^{2j} u)(x) = 0, \tag{1}$$

where ∂_n and ∂_τ are the normal and tangential derivatives on the boundary of the computational domain , respectively, and a_j are coefficients. The parameter J corresponds to the order of the derivatives. If $J < 0$ then (1) is known as the (homogeneous) Neumann boundary conditon, for $J = 0$ and $J = 1$ as Robin or Wentzell boundary condition, respectively. Prominent examples of symmetric local boundary conditions are the Bayliss-Turkel-Gunzberger (BGT) conditions up to order 2 and Feng's conditions at any order for time-harmonic scattering problems.

If only second derivatives are present, *i.e.*, for the Neumann, Robin and Wenttzel conditions, and the boundary is smooth enough, we can incorporate the condition in usual piecewise continuous finite element methods. After J-times integration by parts the variational formulation contains terms like

$$a_j \int_\Gamma \partial_\tau^j u \, \partial_\tau^j v \, \mathrm{d}\sigma(x), \tag{2}$$

and the natural space is $V^J := H^1(\Omega) \cap H^J(\Gamma)$ and positivity of a_j, $j = 0, \ldots, J$ implies well-posedness [6] (for the Helmholtz equation except for a countable set of frequencies). For $J \geq 2$, the usual finite element spaces are not any more contained in V^J. In this case, trial and test functions with $C^{(J-1)}$-continuity (at least) along Γ [3, 4] or auxilliary unknowns [2] may be used.

Following [5] we propose as an alternative nonconforming interior penalty finite element methods for usual continuous finite element spaces of at least order $J-1$, in which for each $j \geq 2$ additional terms on the nodes of the boundary Γ appear. For fourth order PDEs a similar approach has been introduced by Brenner and Sung [1]. We will present well-posedness results and *a-priori* h-convergence error estimates for uniform polynomial degrees. The theoretical convergence results are validated by a series of numerical experiments.

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