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MODEL ORDER REDUCTION USING A MULTILEVEL SCHEME FOR NONLINEAR TRANSIENT HEAT TRANSFER PROBLEM

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Multilevel schemes [1] can be employed in nonlinear problems to avoid computationally expensive iterative solution techniques. In such schemes, the nonlinear terms are replaced by extrapolated values to get a linear system of algebraic equations. The objective of the present study is to investigate the application of model order reduction technique (MOR) [2] to multilevel schemes for a nonlinear finite element analysis of transient heat transfer problem. The coupled nonlinear ordinary differential equations obtained by the spatial discretization of the governing weak form using the standard finite element method are given by [3],

$$\dot{\mathbf{T}} + \mathbf{A}\mathbf{T} = \mathbf{f}(t, \mathbf{T}) \tag{1}$$

where, $\mathbf{A} = \mathbf{C}(\mathbf{T})^{-1}(\mathbf{K}(\mathbf{T}) + \mathbf{H}(t) + \mathbf{R}(\mathbf{T}))$, $\mathbf{f} = \mathbf{f}_1(t) + \mathbf{f}_2(\mathbf{T})$ and $\mathbf{C}(\mathbf{T})$, $\mathbf{K}(\mathbf{T})$, $\mathbf{H}(t)$ and $\mathbf{R}(\mathbf{T})$ have their usual meaning. Here $\mathbf{A}, \mathbf{T}$ and $\mathbf{f} \in \mathbb{R}^N$ and N is number of degrees of freedom (Dof). Discretizing Eq. (1) using a second order accurate multilevel scheme (Dupont II) we get [1],

$$(\mathbf{I} + (\frac{1}{2} + \alpha)\Delta t\mathbf{A}_*)\mathbf{T}_{n+2} = (\mathbf{I} - (\frac{1}{2} - 2\alpha)\Delta t\mathbf{A}_*)\mathbf{T}_{n+1} - \alpha\Delta t\mathbf{A}_*\mathbf{T}_n + \Delta t\mathbf{f}_* \tag{2}$$

where $\mathbf{A}_*$ and $\mathbf{f}_*$ are the extrapolated values at $\mathbf{T}_* = \frac{3}{2}\mathbf{T}_{n+1} - \frac{1}{2}\mathbf{T}_n$. Using two different approaches of MOR the above set of N linearized algebraic equations are transformed to m linearized algebraic equations, where $m << N$. The bases of the projection matrix ($\mathbf{\Phi} \in \mathbb{R}^{N\times m}$) for model order reduction are proper orthogonal decomposition (POD) bases and are generated using the method of snapshots [4].

In the first approach (Approach 1), the projection bases are used to project the system of linearized equations (see Eq. (2)) onto a lower dimension space using $\mathbf{T} = \mathbf{\Phi}\mathbf{z}$. Equation (2) in the reduced dimension is then given by,

$$(\mathbf{I}_r + (\frac{1}{2} + \alpha)\Delta t\mathbf{A}_{*r})\mathbf{z}_{n+2} = (\mathbf{I}_r - (\frac{1}{2} - 2\alpha)\Delta t\mathbf{A}_{*r})\mathbf{z}_{n+1} - \alpha\Delta t\mathbf{A}_{*r}\mathbf{z}_n + \Delta t\mathbf{f}_{*r} \tag{3}$$

where, $\mathbf{A}_{*r} = \mathbf{\Phi}^T \mathbf{A}_* \mathbf{\Phi}$, $\mathbf{f}_{*r} = \mathbf{\Phi}^T \mathbf{f}_*$ and $\mathbf{I}_r \in \mathbb{R}^m$. In the second approach (Approach 2), to avoid the computation of the extrapolated matrices at each time step, the reduced extrapolated matrices, $\mathbf{A}_{*r}$ and $\mathbf{f}_{*r}$, (say at state $\mathbf{T}_i, t_i$) are held constant for few time steps. These number of time steps are decided automatically by comparing the current state ($\mathbf{T}$) with state $\mathbf{T}_i$, using $\left(\frac{\|\mathbf{T}-\mathbf{T}_i\|_2}{\|\mathbf{T}\|_2}\right) < \epsilon$. where, ϵ is a prescribed tolerance value. If the current state does not lie in the neighbourhood of state $\mathbf{T}_i$ then the reduced extrapolated matrices are re-evaluated at the current state $\mathbf{T}$. This procedure is continued to get the complete response of the solution.

To check the applicability of these approaches a temperature distribution in a turbine disk with temperature varying thermal properties (conductivity, convection coefficient and specific heat) is obtained for time dependent convection and radiation boundary conditions. The results obtained using the conventional MOR technique and the proposed approaches are presented in Table 1. The first column shows the results obtained using a finite element model in which the model size is 3926 Dof. The second column shows the results using conventional POD reduced order model (see [4]) where the solution is obtained using an incremental and iterative method. The third and fourth columns show the results obtained using proposed approaches. Here the reduced order model size is 30 in the three approaches. It can be seen from Table 1 that the Approach 2 is significantly faster than the conventional FE approach (about 16 times) and the conventional POD approach (about 11 times). The accuracy of the reduced order models are compared by calculating the mean square relative difference (MSRD). where $\text{MSRD}(t) = \frac{\|\mathbf{T}_F(t)-\mathbf{T}_R(t)\|_2}{\|\mathbf{T}_F(t)\|_2} \times \frac{1}{\sqrt{N}}$. In terms of accuracy the conventional POD reduced order model is most accurate. However the error in the proposed approaches is not very significant.

Table 1: Comparison of computational time and accuracy of different models

	Full FE Model (incremental and iterative method)	POD Model (incremental and iterative method)	Approach 1 reduced order model	Approach 2 reduced order model
Model size/ Bases	3926	30	30	30
Time for solution	3047 sec	2143 sec	557 sec	182 sec
Maximum MSRD	-	0.05%	0.1%	0.3%

Methods to further increase the computational efficiency and the accuracy of the proposed approaches are being currently investigated.

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