

## A Truss Networked Approach to r-Refinement for Computational Fluid Dynamics

Bevan W.S. Jones<sup>\*1</sup>, Arnaud G. Malan<sup>\*2</sup>, Andrew B. Mowat<sup>\*3</sup> and Jakobus A. van Rooyen<sup>\*4</sup>

<sup>\*</sup>Dept. of Mechanical Engineering, University of Cape Town, Rondebosch, 7701, Cape Town, South Africa, <sup>1</sup>jnsbev005@myuct.ac.za  
<sup>2</sup>arnaud.malan@uct.ac.za  
<sup>3</sup>agbmowat@gmail.com  
<sup>4</sup>rnxjak@myuct.ac.za

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The accuracy of Computational Fluid Dynamic (CFD) simulations are directly dependant on the meshes employed. When considering h- and p-refinement [1,2], solution accuracy is typically proportional to mesh complexity. Therefore, achieving a solution of higher accuracy comes with an increase in computational cost. In addition, h- and p-refinement methods can lead to ill balancing when considering parallel computing [3] while it may be challenging to ensure conservation for transient problems. This highlights the need to develop techniques which increase solution accuracy without effecting mesh complexity i.e. r-refinement or optimizing mesh spacing. Further such methods can be used to track transient phenomena, such as gusts or sloshing interfaces in a straightforward manner. As a result, r-refinement is currently receiving increased attention [4,5,6,7].

Repositioning nodes is difficult to do a priori when the flow field is not well known. However, if done concurrently with the solution process, mesh adaptation in conjunction with a monitor function can be used to improve vertex positioning in the interest of solution accuracy. In addition to being driven by an error estimator, redistribution of nodes is to be supported by field equations. Many have used FEM methods [5,6] or Transformations [7]. However, for this project a truss analogy was proposed. The primary difficulty with the truss analogy is robustness and ability to produce a valid mesh after adaption. This is particularly true when nodes are forced into small spaces locally which can result in element collapse. To prevent this, we employed the Ball-Vertex (BV) method [8,9]. Further, mesh quality is maintained by selecting the spring constants as  $1/l$ , where  $l$  is the length past which collapse would occur. The BV method has to date not been employed for the purpose of r-refinement.

The proposed r-refinement methodology is evaluated by application to 2D and 3D benchmark problems. It is shown to be both efficient and robust, while achieving significant improvements in solution accuracy.

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