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PLATE BENDING ANALYSIS BY A MULTI-SCALE MODEL COUPLING BEM AND FEM, CONSIDERING DIFFERENT BOUNDARY CONDITIONS FOR THE RVE

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A multi-scale modelling for analyzing the bending problem of plates composed by heterogeneous materials is presented. The macro-continuum is modeled by the non-linear formulation developed in [1] of the boundary element method (BEM) taking into account the consistent tangent operator (CTO) and based on Kirchhoff's theory. The micro-scale is represented by the RVE (representative volume element) being its equilibrium problem solved by the finite element formulation presented in [2, 3] that takes into account the Hill-Mandel Principle of Macro-Homogeneity while the volume averaging hypothesis of the strain and stress tensors is used to make the micro-to-macro transition. The microscopic equilibrium problem consists of, given the history of the macroscopic strain tensor, finding the field of displacement fluctuation such that, for each instant t, the RVE equilibrium equation is satisfied.

In the numerical example a beam (defined as a narrow plate, see figure (1)) subjected to simple bending is analysed where is adopted a RVE with a void defined in its central (see figure (2a)). Note that despite of being thick, a beam can be analysed with the Kirchhoff's theory as already shown in [4]. The two small sides of the plate are adopted simply supported and the two others, in the span direction, assumed to be free. Its geometry is given by: thickness t=20.0cm, width b=50.0cm, length ℓ=200.0cm and an uniform distributed load g=1kN/cm^2 is applied over all plate domain. The plate boundary was discretized into 16 quadratic elements, while the domain moments are approached over 24 cells, for which the definition of 5 internal points is required as shown in Figure (1b). For the RVE, which is adopted square which length side equal to 1cm, 220 elements and 126 nodes have been used to discretize its domain (see Figure 2a). The material properties over the RVE are: Von Mises elasto-plastic criterion with isotropic hardening k=2000 kN/cm^2, Poisson's ratio ν=0.3; Young's modulus E=20000 kN/cm^2, yield stress σ_y=40.0kN/cm^2. It is evaluated how the numerical response changes according to the adopted RVE boundary conditions which will be adopted as: (i) linear displacements, (ii) periodic displacement fluctuations and (iii) uniform boundary tractions (see [2, 3] for more details). The results are shown in Figure (2b), where can be observed that the limit load obtained considering uniform boundary tractions is significantly smaller. The displacements related to the three different boundary conditions are very similar for $\beta \leq 0.75$, when the dissipative processes in the microstructure due to presence of the voids and ductile behaviour are not so strong. After that the analysis considering uniform tractions along the RVE boundary did not converged, while for the other two boundary conditions the analysis continued further. Thus the results confirm what had already been verified in other works (see [2, 3]): the linear boundary displacement gives the stiffest (most cinematically constrained) solution while the uniform boundary traction model produces the most compliant (least kinematically constrained).

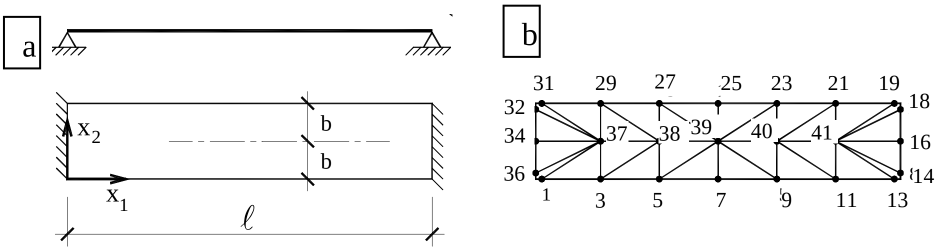

Figure 1 – Simply supported beam – geometry and discretization

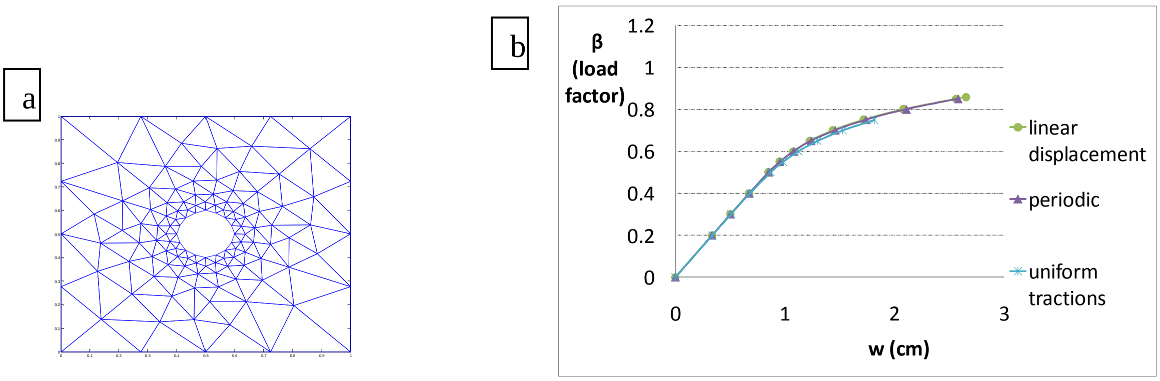

Figure 2 – a)Discretization of a RVE with a void defined in the centre b)Deflection at the central point of the beam, using different boundary conditions in the RVE

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