A ROBUST NITSCHE’S FORMULATION FOR INTERFACE PROBLEMS WITH SPLINE-BASED FINITE ELEMENTS

Wen Jiang¹, Chandrasekhar Annavarapu², John E. Dolbow³ and Isaac Harari⁴

¹ Graduate student, Department of Mechanical Engineering and Material Science, Duke University, Durham, NC 27708, U.S.A
² Postdoctoral scholar, Atmospheric, Earth, and Energy Division, Lawrence Livermore National Laboratory, Livermore, CA 94550, U.S.A
³ Professor, Department of Civil and Environmental Engineering, Duke University, Durham, NC 27708, U.S.A
⁴ Professor, Faculty of Engineering, Tel Aviv University, 69978 Ramat Aviv, Israel

Key words: B-Splines, X-FEM, Nitsches method, embedded interface, Stabilized.

The extended finite element method (X-FEM) has proven to be an accurate, robust method for solving embedded interface problems. X-FEM is mostly used with linear approximation shape functions and linear level set functions. Our purpose is to accelerate the convergence of X-FEM using higher order shape functions for the finite element approximation. Smooth functions such as rational B-splines are becoming increasingly popular for use in finite element approximations to solutions of partial differential equations. In the current work, the X-FEM formulation is incorporated into spline-based finite elements based on the Nitsche technique for enforcing constrains. The geometric representation of curved interface has a great influence on the accuracy. We employ a hierarchical local refinement approach to improve the geometrical representation of the curved interface. With this approach, optimal rate of convergence is achieved while keeping a linear representation of the geometries.

In this work, we further propose a novel weighting for the interfacial consistency terms arising in a Nitsche variational form with higher order basis function. The choice of the weighting has a great bearing on the stability of the method. A qualitative dependence between the weights and the stabilization parameter is established in such a way that additional element level eigenvalue calculation is performed. An important consequence of this weighting is that the bulk as well as the interfacial fields remain well behaved in the presence of (a) elements with arbitrarily small volume fractions, (b) large material heterogeneities and (c) both large heterogeneities as well as arbitrarily small elements. We demonstrate the accuracy and robustness of the proposed variational form through several numerical examples and contrast it against classical Nitsche’s form.