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A NEW HIGH-ORDER SPATIAL GALERKIN DISCRETIZATION BASED ON FOURIER CONTINUATION METHODS

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It is well-known that the classical numerical methods widely applied in linear and nonlinear dynamics, such as finite difference or finite element methods (even when their high-order versions are considered), suffer from numerical dispersion due to the local character of their approximation function basis. Despite of the advantage of the sparsity of the discrete matrices involved in this kind of methods, and the recent advances of enriched discrete methods to mitigate the numerical pollution, the accuracy to approximate the solution of wave propagation problems is limited to middle frequency range.

To remedy those drawbacks related to the numerical dispersion, spectral methods based on Chebyshev polynomials or trigonometric polynomials are also widely used [1]. However, the application of the Chebyshev procedures require special refined grids close to the boundary of the computational domain, they are limited to simple geometries (or it requires the knowledge of mappings from the original domain into a simple geometry), or what can be more restrictive in the case of Fourier methods, they can be only applied to problems with periodic solution, otherwise, the Gibbs phenomena would perturb the accuracy of the spectral approximation.

Recently, special attention has been focused on Fourier Continuation (FC) methods [3, 4] due their ability of transforming arbitrary partial differential equations (PDE) stated in a general geometry setting into a periodic framework where, using a standard Fourier discretization procedure, the constant-coefficient differential operators can be diagonalized in closed form and classical efficient computational techniques, such as the Fast Fourier Transform, can be applied accurately.

These FC methods have been already applied to the discretization of different models, such as the Navier-Stokes equations [2] or heterogeneous diffusion and wave-like models

[6]. In both cases, the Fourier discretization was based on a collocation procedure in a Cartesian grid, where the non-homogeneous boundary conditions were imposed explicitly by means of a linear combination of solutions of the model in closed-form [4] or computed using a numerical approximation [6].

In the present work, a new approach for the FC discretization of diffusion and wave-like models is introduced using a Galerkin procedure based on a trigonometric polynomial discretization. In this new approach, to compute the Fourier continuation (i.e. to compute the Fourier amplitudes of a trigonometric polynomial) associated to the source term in the discretization of a PDE model, it is necessary to compute the solution of a linear system involving an hermitian Toeplitz matrix. In a grid of N points, the solution of the linear system requires $O(N \log^2(N))$ computations using fast randomized algorithms [7], which is a similar computational cost to other FC procedures (see for instance [5]). The treatment of the boundary conditions are handled by using a Lagrange multiplier technique, which introduces the boundary conditions as additional constrains in the discrete Fourier problem.

Finally, some numerical results illustrating the robustness and accuracy of the method are shown. In addition, the advantages of the proposed method are also discussed comparing the present Galerkin method with those results obtained with the previous collocation methods available on the literature.

REFERENCES

[1] J.P. Boyd. *Chebyshev and Fourier spectral methods.* Dover Publications, 2001.

[2] O.P. Bruno and N. Albin. A spectral FC solver for the compressible Navier-Stokes equations in general domains I: Explicit time-stepping. *J. Comput. Phys.*, Vol. **230**, n. 16, 6248–6270, 2011.

[3] O.P. Bruno, Y. Han, and M.M. Pohlman. Accurate, high-order representation of complex three-dimensional surfaces via Fourier continuation analysis. *J. Comput. Phys.*, Vol. **227**, n. 2, 1094–1125, 2007.

[4] O.P. Bruno and M. Lyon. High-order unconditionally stable FC-AD solvers for general smooth domains I. Basic elements. *J. Comput. Phys.*, Vol. **229**, n. 6, 2009–2033, 2010.

[5] M. Lyon. A Fast Algorithm for Fourier Continuation. *SIAM J. Sci. Comput.*, Vol. **33**, n. 6, 3241–3260, 2011.

[6] O.P. Bruno and A. Prieto. Spatially Dispersionless, Unconditionally Stable FC-AD Solvers for Variable-Coefficient PDEs *J. Sci. Comput.*, Vol. **58**, n. 2, 331–366, 2014.

[7] J. Xia, Y. Xi, and M. Gu. A Superfast Structured Solver for Toeplitz Linear Systems via Randomized Sampling. *SIAM J. Matrix Analysis Applications*, Vol. **33**, n. 3, 837–858, 2012.