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BIFURCATION ANALYSIS OF THERMAL CONVECTION AT FINITE PRANDTL NUMBER IN NON-ROTATING SPHERICAL SHELL

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Rayleigh-Bénard convection in a couple of spherical boundaries has attracted much attention for over a hundred years due to its relevance to geophysical tectonic movement and magnetic field sustaining process under the earth's crust[1]. Fig. 1 shows the configuration of concentric dual spherical boundaries with radiuses, r_{in} and r_{out}, in which Newtonian fluid with a constant volumetric heat production rate, β, the positive thermal expansion coefficient, α, and kinematic viscosity, ν, is confined. The convection can be induced by the opposing effects on the fluid of spherically centripetal gravity and centrifugal buoyancy. Such a convection driven at high Rayleigh number could be even a model describing motion in the outer core of the earth.

Investigating stability of axisymmetric thermal convective states, in case of non-rotation and infinite Prandtl number, Zebib and others[2] reported that six different states of steady convection can be realised around the critical Rayleigh number. In the present work, we perform direct numerical simulation of thermal convection in terms of Galerkin method so as to pursue structural change of these stable states from the infinite to finite Prandtl number, $Pr = 1$. Provided that all fluid properties except for density are constant in the context of the Boussinesq approximation and that the spherically centripetal gravitational acceleration is proportional to the distance from the centre (γ is a constant of proportionality), the following nondimensional governing equations describe the time development of our system;

$$
\begin{aligned}
\boldsymbol{\nabla} \cdot \boldsymbol{u} &= 0 \ , \\
\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} &= -\boldsymbol{\nabla} p + Ra\Theta \boldsymbol{r} + \boldsymbol{\nabla}^2 \boldsymbol{u} \ , \\
Pr(\partial_t \Theta + \boldsymbol{u} \cdot \boldsymbol{\nabla}^2 \Theta) &= \boldsymbol{r}\boldsymbol{u}_r + \boldsymbol{\nabla}^2 \Theta \ ,
\end{aligned}
$$

where non-slip and isothermal boundary conditions are imposed at the boundaries for velocity $\boldsymbol{u}$ and temperature disturbance Θ. It should be noted that the equations are

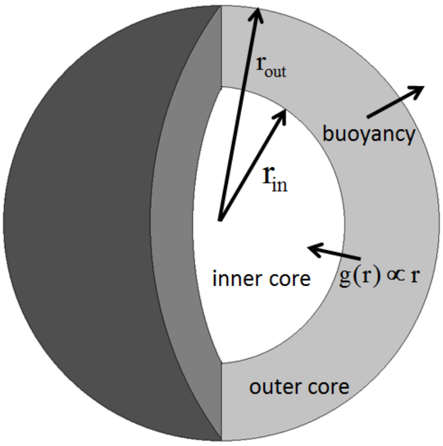

Figure 1: Newtonian fluid is confined between concentric dual spherical boundaries with radiuses, r_{in} and r_{out}. A variety of thermal convective states may emerge from the static state via a supercritical bifurcation owing to spherically centripetal gravity and centrifugal buoyancy over the critical Rayleigh number.

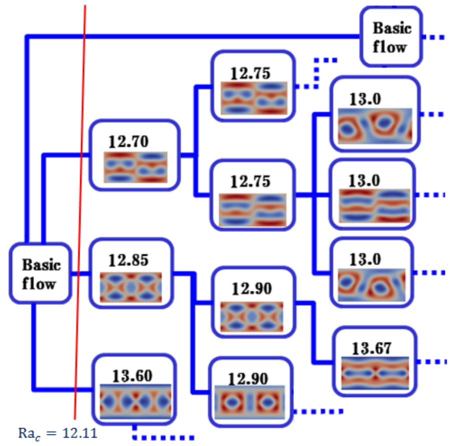

Figure 2: Bifurcation diagram around the critical Rayleigh number, $Ra_c = 12.1$, for $\eta = 0.5$. Principally, our numerical simulation traces only stable states, which are illustrated by contours of radial direction velocity, u_r, at the intermediate radius, $r = (r_{\text{in}} + r_{\text{out}})/2$, in Mercator projection.

determined only by the three dimensionless parameters, Rayleigh number, Ra, Prandtl number, Pr and ratio of radiuses, η,

$$Ra = \frac{\alpha\beta\gamma d^6}{\nu\kappa}\ , \quad Pr = \frac{\nu}{\kappa}\ , \quad \eta = \frac{r_{\text{in}}}{r_{\text{out}}}\ ,$$

where κ and d are thermal diffusion coefficient and the characteristic length of the system, respectively. Half the distance of boundaries, $(r_{\text{out}} - r_{\text{in}})/2$, is adopted as characteristic length, so the critical Rayleigh number is $Ra_c = 12.1$ for $\eta = 0.5$ where the static state loses its stability and simultaneously bifurcates to three different convective states, because of the multiplicity of spherical harmonics as to the instability of the static state [3]. Tremendous numerical efforts have been devoted to quest stable steady states, so that we found that these convective states successively bifurcate to their "offsprings", which are illustrated in Fig. 2 as a family tree.

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