

# ON THE STRONG FORM CLOUD-BASED FLUX-FREE IMPLICIT RESIDUAL ERROR ESTIMATORS FOR $C^k$ -GFEM APPROXIMATIONS

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An implicit residual error estimator for smooth GFEM-based approximations [1] is presented. The purpose of the work is to investigate some advantages of continuous Partition of Unity (PoU) functions for error estimation in terms of energy norm through computation of nodal error indicators following subdomain-based procedures [2–5]. Aspects as construction of  $C^k$ -GFEM based approximations functions using domain triangulation [6] are addressed and it is shown how the higher regularity may be exploited in implicit residual algorithms for error estimation. The approximated stress fields can be directly differentiated in order to verify the equilibrium equations in strong form, leading to a continuous residuum field. The variational representation of the error on the clouds involves bilinear forms in terms of the estimated local error similar to those strategies presented in [4] and [5] and the selection of higher-order enrichment functions is similar to those proposed in [2]. On the other hand, such residuum field may be projected over the set of higher-order functions resulting in the excitation for the variational error problems posed on the clouds, considering the localization by the PoU [3]. This innovative way of computing the linear functional therefore considers higher-order derivatives of the approximated solution through the residuum field. The construction of local problems on the clouds through localization using the PoU property naturally nullifies any contribution of non-homogeneous Neumann boundary conditions. The smoothness of functions also renders continuous error approximations inside each cloud. Additionally, rigid body motions are prevented due to the nature of the enrichment functions considered. Even though the computable version of the upper bounds for the estimators are not guaranteed by definition, local effectivity for the nodal indicators has been obtained around singularities in the cases studied so far. Moreover, as there are not stress jumps at cloud boundaries due

to the smoothness, different ways of localization could be envisioned, leading to the incorporation of non-homogeneous Neumann boundary conditions directly computed from the solution and not requiring equilibrating process.

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