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PGD method with material non-linearities with enthalpic approach applied foundry

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1. Introduction

PGD method (Proper Generalised Decomposition) is based on separated representation of unknowns, for instance in time and space, and on an iterative method to solve equation system. Ammar [1] and Pruliere [6] have already proposed particular resolutions for specific non linearities. We propose to generalize resolution with non-linear material properties, by discretising them in time and space. This approach is applied to solve the heat equation written in enthalpy.

This study is done in the context of the French Region Picardie Project ProFond which include two industrial partners (Montupet SA et E.J.) leaders in foundry.

2. Methods

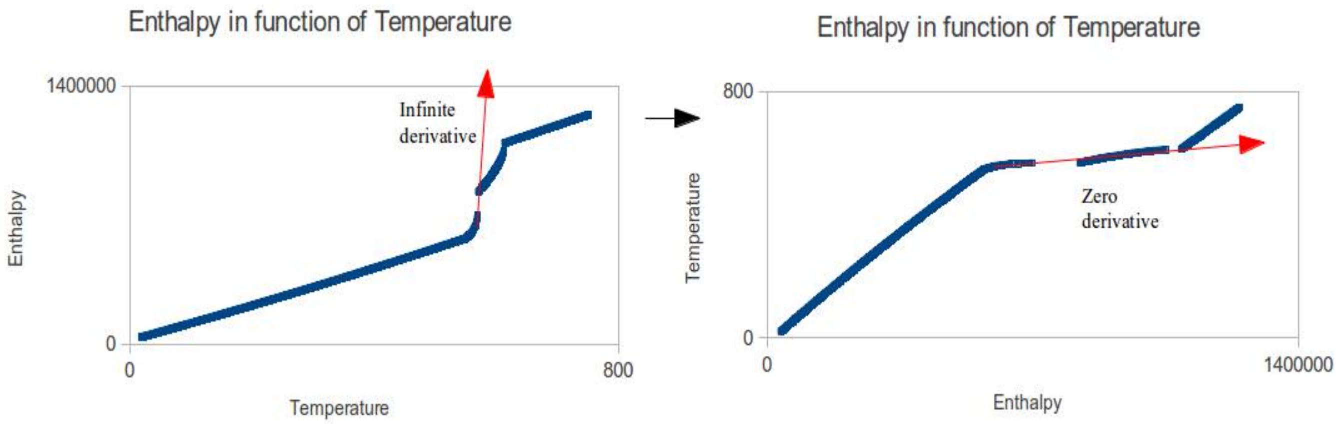
We offer to solve heat equation in enthalpy by solving *H* as proposed by Nedjar [5] and later Feulvarch [3]. Material properties, *ρ, k,* c_p and the solid fraction *fs,* depends on temperature, itself depending on space and time : $T(x,t)$.
Phase change and release of latent heat give lot of trouble in numerical resolution, in particular in the infinite derivatives of material properties. The approach developed in [3] rewrites the heat equation so that we get two equations with two unknowns (H,T) :

$$\frac{\partial H}{\partial t} - \mathrm{div}(k\,\mathbf{grad}\,T) = 0 \quad \text{and} \quad T = g(H) \qquad (1)$$

The fields temperature *T* and enthalpy *H* are defined on space time Ω x [0,t_{end}], and the function *g(H)* is supposed to be known, and its derivative exists for all *H*.

We see that temperature is related to enthalpy by a constitutive relation coming from metallurgical material properties. Figures (a) and (b) are coming from following enthalpic expression : $H = \int \rho c_p dT + \rho L(1 - f_s)$.

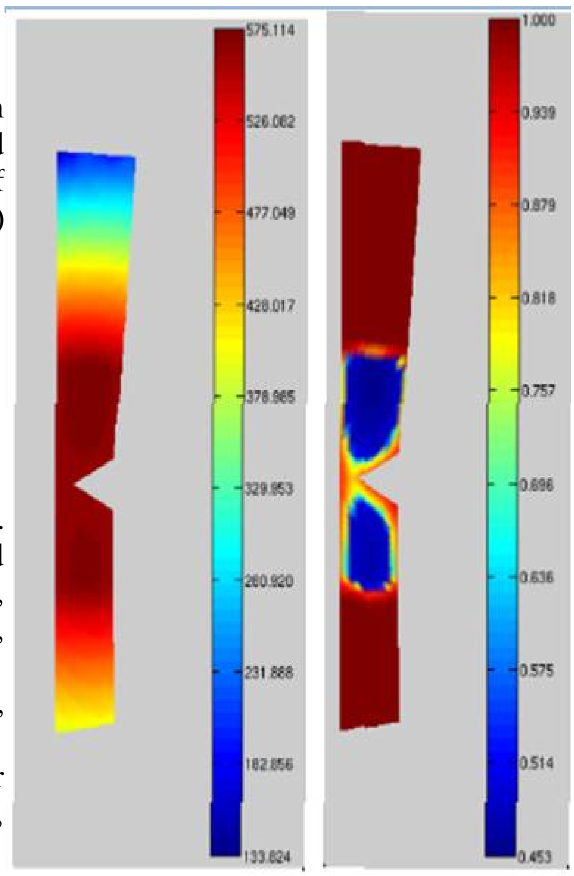
To solve equation (1) and (2) by the PGD method, we have to introduce separated representation. Joyot [4] propose to do a POD (Proper Orthogonal Decomposition) on each property to obtain a separation of variables. This way is time and memory expensive.
So, we offer to separate variables via a finite element discretisation through space-time elements, e.g. conductivity could be rewritten $k(T(x,t)) = N(x) * [K] * P(t)$ with shape function *N(x)* et *P(t)*, respectively space et time.



(a) *(b)*

This enthalpic approach gives us an other way to code PGD with non-linearity materials.

(c) *(d)*



3. Results

The sample, in aluminium alloy, have been modelled with an initial temperature 720°C and convective flux on the edges. Results in term of temperature T (°C) (Fig c) and solid fraction fs (Fig d) are displayed at a ponctual instant.

4.References

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