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LOCKING-FREE INTERIOR PENALTY METHODS FOR ELASTICITY PROBLEMS, USING QUADRILATERAL ELEMENTS

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In the context of elasticity problems, interior penalty (IP) discontinuous Galerkin methods have in the past been shown to be uniformly convergent in the incompressible limit, thus circumventing the pathological behaviour known as "locking" which is exhibited by the standard Galerkin finite element method. However, analyses for the primal IP methods that take material parameters into consideration have been limited to meshes of simplices (e.g. [1], [2], [3]).

We show, firstly, numerical evidence that with three well-known IP methods for linear elasticity, bilinear quadrilateral elements perform poorly in contrast to linear triangular elements, in particular showing locking-type behaviour.

Secondly, we present an *a priori* error analysis for the IP methods with bilinear elements (considering the three IP methods in a single general formulation). Challenges involved in extending a convergence analysis for triangular elements to one for quadrilaterals are considered. One critical aspect of the proof is the choice of interpolant to be used for splitting the approximation error. A new interpolant, in conjunction with a novel approach to splitting the error, provides the necessary properties for the proof, for rectangular elements. The analysis shows that, as indicated by the numerical results, the approximation error depends undesirably on the material parameters.

Finally, we present, as a remedy for this problem, a modified IP formulation for bilinear (or trilinear) elements in which selected edge terms (or face terms, in three dimensions) are under-integrated. An *a priori* analysis for this new formulation shows optimal uniform convergence with respect to the compressibility parameter. Numerical results illustrate the good performance predicted by the theory.

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