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TIKHONOV REGULARIZATION FOR THE MODIFIED MAPPING-COLLOCATION METHOD APPLIED TO CIRCUMFERENTIAL CRACK IN A CURVED BEAM

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In the present study, zeroth-order Tikhonov regularization is successfully used to resolve the numerical difficulties associated with application of the modified mapping collocation (MMC) method to the problem of a circumferential crack in an isotropic curved beam under pure bending. The target of analysis is to obtain accurate values of stress intensity factors and thereafter energy release rate. *Tikhonov-regulated MMC method* results are compared with the values obtained by the finite element analysis (ABAQUS results from [1]) for a sample case. It is shown that there is a good agreement in terms of energy release rate and mode mixity. The regularization process enables increasing the number of stations and consequently series order and makes convergence study possible.

The boundary value problem of a circumferential crack in an isotropic curved beam under pure bending using the MMC method has been addressed by the authors recently [2]. Lack of an analytical solution in general series form (like Westergaard solution for stright crack) makes the solution for a curved crack more vulnerable numerically. The problem is ill-posed. For a system with a rank-deficient or very nearly singular coefficient matrix, least squares method often gives solutions that vary unstably. Tikhonov regularization is a numerical procedure which aims at stabilizing the systems suffering ill-posedness.

The MMC method is a semi-analytic approach that combines complex analysis tools (mainly based on Muskhelishvili formulation) with boundary collocation method. *Muskhelishvili potential* is expanded in a complex Laurent series form. The boundary condition equations are written for discrete points (stations) on the boundary. An overdetermined linear system of boundary condition equations is formed which its unknowns are coefficients of the series expansion. The coefficient matrix of the overdetermined system is very dense (virtually has no zeros) and is also ill-conditioned. These characteristics limits the power of the MMC method to some ranges of geometrical parameters and also confines

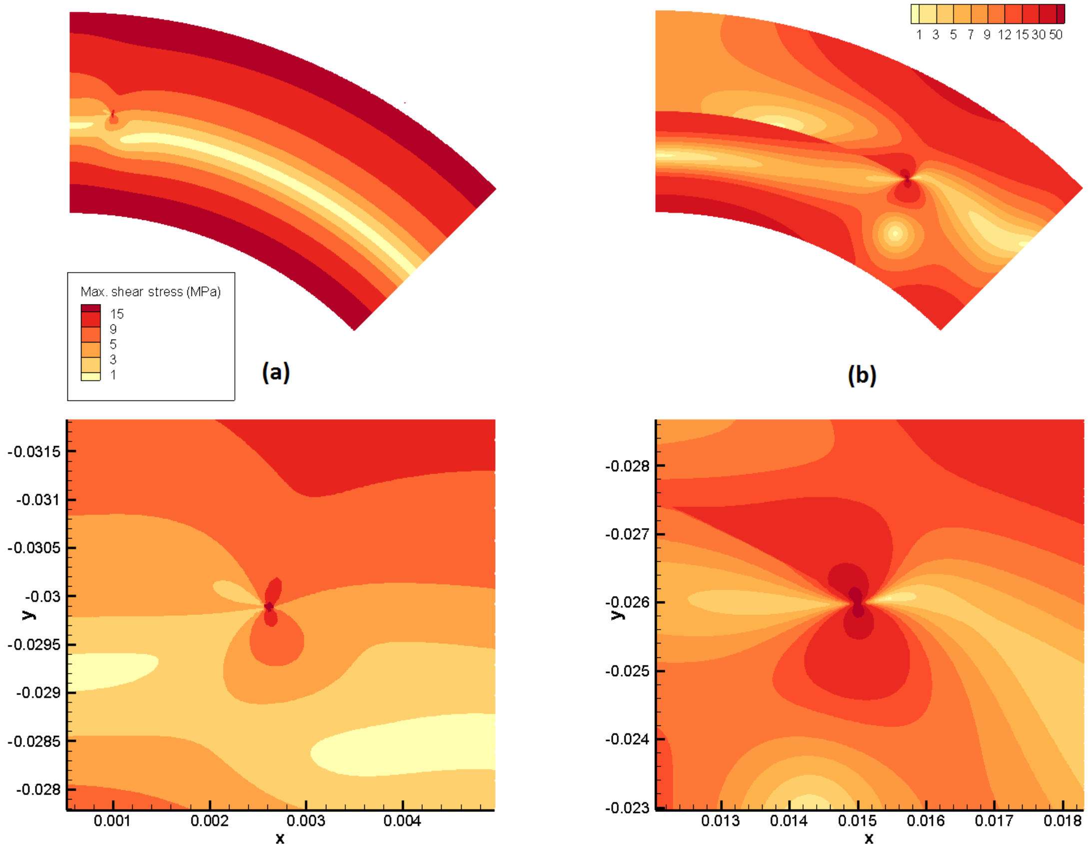

Figure 1: Contours of constant τ_{max} with the corresponding zoomed view of the crack tip obtained by Tikhonov-regulated MMC method for two different crack half arc angles **(a)** $\beta = 5°$ and **(b)** $\beta = 30°$, for the the sample case $R_1 = 24\ mm$, $R_2 = 36\ mm$ and $R_a = 30\ mm$.

the order of the series expansion which sometimes even hinders doing convergence tests. Accurate computation of the series coefficients is crucial in determining the stress field and thereafter stress intensity factors. Tikhonov regularization method appears very helpful in settling the mentioned numerical barriers.

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