A differential structured-based model based on stochastic evolution equations for the scalar turbulence parameters

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Engineering analysis of complex turbulent flows relies heavily on turbulence models. Current conventional turbulence models work well in near-equilibrium situations where turbulent stresses can be predicted through algebraic expressions based on an eddy viscosity assumption (EV). In complex 3D flows and under non-equilibrium conditions the performance of these models is in general not satisfactory. This deficiency is mainly attributed to key physics that are discarded as a result of the simplistic EV assumption. Thus, more elaborate classes of models have been used to account for non-equilibrium features, such as Differential Reynolds Stress Models (DRSM). However, DRSMs have in general failed to show a consistent superiority in dealing with complex turbulent flows and thus have not been widely adopted in industry.

One major reason for this is that DRSMs depend only on the Reynolds stresses, whereas Kassinos and Reynolds [1] have shown that additional one-point tensors that describe turbulence eddy morphology are needed in order to provide a complete description of turbulence. The need to incorporate additional structural information motivated Kassinos and Reynolds [1] to introduce a new class of models, called structured-based models (SBMs), which are directly or indirectly based on the exact transport equations for the one-point turbulence structure tensors.

Direct use of the transport equations is made in differential SBMs, such as the Q-model that solves directly the transport equations of all structure tensors that characterize the morphology of turbulence. While Q-model has been shown promising performance in flows with strong rotational effects, the need to solve 27 equations renders it less attractive as an engineering tool. Thus, a second type of differential SBM was constructed that is based on the transport equation for the eddy axis tensor $a_{ij}$. This simplified and computationally attractive framework captures the key phenomenology of the exact structure tensor transport equations.

In addition to $a_{ij}$, two scalar parameters are evolved that determine the character of the turbulence structure: $\phi$ is the fraction of energy in the jetal mode (motion along the eddy axes), $1-\phi$ is the fraction of energy in the vortical mode (motion in the plane normal to the eddy axes) and $\gamma$ is the jet-vortex correlation parameter. Structure tensors are obtained through non-linear algebraic expressions relating them to the structural parameters. Although the SBM model produced excellent predictions in the RDT limit, matching RDT for basically all mean deformation modes, it performed poorly for homogeneous flows subjected to slow irrotational deformations. Kassinos et al. [2] attributed these difficulties to the algebraic constitutive equations that were used. For slow deformations, non-linear interactions should yield non-zero $\gamma$ and $\phi$, which was not feasible given the current form of the algebraic
constitutive equations. Thus, in order to capture these effects, modifications to the transport equations are needed, which must also preserve the full realizability that SBM model currently enjoys.

The above considerations motivated the current work, in which modified expressions for all transport and algebraic expressions have been developed in a way that ensures the model’s full realizability. The new model gives excellent results in the RDT limit, matching original model. For slow mean deformations, stochastic processes were introduced that give non-zero values for γ and φ, in agreement with physical arguments. The predictions of the improved and original models for three basic cases of slow irrotational mean deformation are shown in Figure 1. The evolution of the turbulent stress anisotropies is compared to DNS data showing that the new model gives improved results, which is significant given the computational attractiveness of this differential SBM model.

![Figure 1](image1.png)

Figure 1: Model predictions for the evolution of the Reynolds stress anisotropies with total strain (C) using the current model (solid line) and original model (dashed line) compared to DNS results (symbols); (a) axisymmetric expansion (Sk₀/ε₀=0.41), (b) plane strain (Sk₀/ε₀=0.50) and (c) axisymmetric contraction (Sk₀/ε₀=0.41).

REFERENCES
