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# NONLINEAR KIRCHHOFF-LOVE SHELLS: THEORY AND NUMERICAL ASSESSMENT USING TUBA FINITE ELEMENTS

**V. Ivannikov[1], C. Tiago[1] and P. M. Pimenta[2]**

[1] Instituto Superior Técnico, Universidade de Lisboa,
Av. Rovisco Pais, 1049-001 Lisboa, Portugal
{vladimir.ivannikov,carlos.tiago}@civil.ist.utl.pt
[2] Escola Politécnica da Universidade de São Paulo,
P.O. Box 61548, 05424-970, São Paulo, SP, Brasil,
ppimenta@usp.br

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The current work discusses the application of the TUBA finite elements [1,2], initially developed for thin plates problems, to geometrically exact thin shell analysis.

The proposed geometrically exact shell model is derived from the 3D continuum mechanics. The shell kinematics is based on the Kirchhoff-Love assumption and is characterized by the deformation gradient, which yielded the generalized cross-section strain measures — stretches and curvatures, written in terms of first- and second-order derivatives of displacements [3]. As the energetically conjugate quantity, the first Piola-Kirchhoff stress tensor is chosen. The shell's initial geometry is exactly represented using a mapping from a reference configuration. A neo-Hookean material functional, supplemented by the plane stress condition, is incorporated in the constitutive level of the model.

The theory is derived independently from the specific numerical method to be used, thus allowing any available approximation of $C^1$ type to be used for the implementation. Special attention is given to the treatment of the natural and essential boundary conditions.

The TUBA family of plate finite elements is considered for the discretization of the displacement vector field. These triangular elements, with necessarily straight sides in the reference configuration, provide $C^1$ continuous approximations, mandatory for the proper implementation of the Kirchhoff-Love model. The family contains 3 pairs of elements (primary and reduced), which provide polynomial approximants of degrees $k = 5,\ 6$ and 7 respectively. The reduced elements have a simpler structure and a smaller number of nodes, but still guaranty $C^1$ continuity [4]. The boundary normal derivative of the approximation in these elements is constrained, such that its order is reduced by one unit, from $(k-1)$, as for the primary elements, to $(k-2)$.

Due to complexity of the elements degrees of freedom (which are the function value, its first- and second-order derivatives), the imposition of the essential boundary conditions is not trivial and needs some preliminary analysis of the boundary behavior.

The proposed model is assessed by means of numerical nonlinear shells problems examples.

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