

## STEADY RISE OF A DEFORMABLE BUBBLE IN AN ELASTO-VISCOPLASTIC FLUID

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The singularity arising at the yield surface when the Bingham or other viscoplastic models are used has been one of the major problems in all efforts to simulate viscoplastic fluids. Different methods have been proposed as a way to avoid this problem, including regularization of the constitutive models or the augmented Lagrangian method [1, 2]. Despite this progress, model predictions often deviate from experimental measurements suggesting that material elasticity and thixotropy must be accounted for in the constitutive model [3, 4]. To this end, we have revisited the challenging problem of the steady rise of a bubble due to buoyancy. We examined the predictions of the regularized viscoplastic model proposed by Papanastasiou [5], which is valid both in the yielded and the unyielded part of the material when it is coupled it with the exponential PTT viscoelastic model to account for fluid elasticity following the recent ideas reported in [6, 7].

The bubble response depends on the following parameters: Bond  $Bo = \rho^* g^* R_b^{*2} / \gamma^*$ , Archimedes  $Ar = \rho^{*2} g^* R_b^{*3} / \mu_o^{*2}$ , Deborah  $De = \lambda^* \rho^* g^* R_b^* / \mu_o^*$ , Bingham  $Bn = \tau_y^* / \rho^* g^* R_b^*$ , where  $\rho^*$  is the density,  $\mu_o^*$  the viscosity,  $\lambda^*$  the elasticity and  $\gamma^*$  the surface tension of the material,  $g^*$  is the gravitational acceleration,  $R_b^*$  the radius of a spherical bubble of the same volume and  $\tau_y^*$  is the yield stress of the material. The very complex set of PDEs (momentum and mass balances along with a complex constitutive law in two dimensions, assuming axial symmetry) is solved with appropriately modified Finite Element method. We use the SUPG method for the weighting of the constitutive equations along with the EVSS-G splitting method for the computation of the elastic stresses and a local mesh refinement around specific areas of the bubble. We first tested our new numerical algorithm against earlier results for viscoplastic [1, 2] and viscoelastic fluids [8], where bubbles are known to develop a long tail, a jump in their rise velocity and hysteresis phenomena and found quantitative agreement. Next we examined the predictions of the new elasto-viscoplastic model, which is valid both in the yielded and the unyielded part of the material. The results of this new model predict bubble shapes with a cusp in the backside of the bubble, which has been observed in experiments with carbopol solutions [4]. As expected, fluid plasticity moderates the elastic effects and introduces a yield surface at a finite distance around the bubble. This yield surface

approaches the bubble as the Bingham number increases leading to bubble entrapment in the liquid. Unyielded material arises around its equatorial plane under certain parameter values. A complete parametric analysis will be presented along with comparisons with published experimental results.

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