ON THE FORWARD IN TIME SOLUTION OF THE UNSTEADY ADJOINT EQUATIONS

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This paper aims at overcoming the significant memory requirements and/or computational cost of solving the unsteady adjoint equations backward in time for the optimization of large scale unsteady problems which require very fine temporal discretization, such as the unsteady turbulent flows modeled by DES/LES/DNS approaches, where efficient approaches such as the checkpointing technique would fail.

The proposed algorithm has two phases. At the first phase, the unsteady adjoint equations are solved backward in time, based on the average in time field of the state variables. The computational cost is almost equal to that of solving the unsteady state equations and requires the additional storage of the average in time state variable field only. An adjoint variable field corresponding to the initial time is computed and stored at the end of this phase.

This initial condition feeds the second phase, the forward solution, whose cost is almost equal to twice that of the unsteady state equations. This is due to the fact that the state variable field at each time step has not been stored at the first phase, but is recomputed at the second phase at each time step, together with the solution of the unsteady adjoint equations. Thus, the total computational cost is almost equal to three times the solution of the unsteady state equations instead of two in the case of the steady state problem, which is relatively low, keeping in mind that there are negligible additional memory requirements, compared to the conventional unsteady adjoint approach.

The proposed forward in time unsteady adjoint approach is compared to (a) the local in time adjoint approach and (b) the approach that is based on a simple averaging of the state variable field, which corresponds to the first phase of the proposed approach. The adjoint solutions are compared to that obtained from the exact backward in time solution of the unsteady adjoint equations.
The comparisons are made on the 1D Burgers equation with a non-smooth in time source term. The adjoint field computed using the proposed algorithm is by far more close to that computed using the conventional backward in time adjoint solution than those computed using the other two approaches.

Figure 1: Distribution over space and time of the state variable (top-left), local in time adjoint variable (top-right), averaging adjoint variable (bottom-left) and the proposed forward in time adjoint variable (bottom-right) in comparison with the exact backward in time adjoint variable.

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REFERENCES
