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# Application of Fractional Continuum Mechanics to Plane Problems of Elasticity

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In this contribution the application of non-local fractional continuum mechanics (FCM) for plane strain and plane stress elasticity is presented. FCM is a generalisation of classical continuum mechanics utilising fractional calculus - a branch of mathematical analysis that studies the differential operators of an arbitrary (real or complex) order. The kinematics and stress concepts are discussed and governing equations in terms of displacements for both plane problems are defined. The numerical implementation utilising generalised finite difference method is shown in detail. Three cases are solved to indicate the role of order of fractional continua and length scale: biaxial tension, pure shear and complex state.

Among other concepts dealing with non-locality resulting from fractional calculus application the presented formulation has the following crucial advantages: (i) the proposed new formulation has clear physical interpretation and is developed by analogy to general framework of classical continuum mechanics; (ii) we deal with finite; (iii) the generalised fractional measures of the deformation e.g. fractional deformation gradients or fractional strains, have the same physical dimensions as classical ones (thus their classical interpretation remain unchanged); (iv) characteristic length scale of the particular material is defined explicitly (as in classical non-local models what is more robust than implicit techniques); (v) objectivity requirements are proved; (vi) and finally, the discussed concept bases on the fractional material and spatial line elements.

Classical (local) solution is obtained as a special case.

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