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NEW DIFFERENTIAL OPERATORS AND DISCRETIZATION METHODS FOR LARGE-EDDY SIMULATION AND REGULARIZATION MODELING

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The essence of turbulence are the smallest scales of motion. They result from a subtle balance between convective transport and diffusive dissipation. Numerically, if the grid is not fine enough, this balance needs to be restored by a turbulence model. The most popular example thereof is the Large-Eddy Simulation (LES). Shortly, LES equations result from filtering the Navier-Stokes (NS) equations in space

$$\partial_t \overline{\boldsymbol{u}} + (\overline{\boldsymbol{u}} \cdot \nabla)\overline{\boldsymbol{u}} = \nu \Delta \overline{\boldsymbol{u}} - \nabla \overline{p} - \nabla \cdot \tau(\overline{\boldsymbol{u}}) \, ; \quad \nabla \cdot \overline{\boldsymbol{u}} = 0, \tag{1}$$

where $\overline{\boldsymbol{u}}$ is the filtered velocity and $\tau(\overline{\boldsymbol{u}})$ is the subgrid stress tensor and approximates the effect of the under-resolved scales, *i.e.* $\tau(\overline{\boldsymbol{u}}) \approx \overline{\boldsymbol{u} \otimes \boldsymbol{u}} - \overline{\boldsymbol{u}} \otimes \overline{\boldsymbol{u}}$. Then, the closure problem consists in replacing (approximating) the tensor $\overline{\boldsymbol{u} \otimes \boldsymbol{u}}$ with a tensor depending only on $\overline{\boldsymbol{u}}$ (and not $\boldsymbol{u}$). Because of its inherent simplicity and robustness, the eddy-viscosity assumption is by far the most used closure model

$$\tau(\overline{\boldsymbol{u}}) \approx -2\nu_e S(\overline{\boldsymbol{u}}), \tag{2}$$

where ν_e denotes the eddy-viscosity and $\tau(\overline{\boldsymbol{u}})$ is considered traceless without the loss of generality. The success of a turbulence model depends on the ability to capture well the above-mentioned (im)balance between the convective transport and the diffusive dissipation. In this regard, many turbulence eddy-viscosity models for LES have been proposed in the last decades. In order to be frame invariant, most of them rely on differential operators that are based on the combination of invariants of a symmetric second-order tensor (with the proper scaling factors). To make them locally dependent such tensors are derived from the gradient of the resolved velocity field, $\mathrm{G} \equiv \nabla \overline{\boldsymbol{u}}$. Namely, the classical Smagorinsky model is based on the second invariant of $\mathrm{S} = 1/2(\mathrm{G} + \mathrm{G}^T)$, the WALE model [1] is based on the second invariant of the traceless part of $1/2(\mathrm{G}^2 + (\mathrm{G}^2)^T)$, the Vreman's model [2] is based on the second invariant of $\mathrm{G}\mathrm{G}^T$, the model proposed by Verstappen [3] is based on the second and third invariants of S and the σ-model [4] is based on the three eigenvalues (invariants) of the GG^T tensor. Apart for the Smagorinsky model, the rest of models switch off ($\nu_e \to 0$) near the solid walls although only the WALE and the σ-model have the proper cubic behavior. Although the lists differ, other desirable properties (*i.e.* $\nu_e \geq 0$, ν_e vanishing for 2D flows and laminar flows,...) are also achieved by these differential operators. In this work, all these models will be presented in a unified framework together with the new proposed differential operators proposed in the context of regularization modeling of turbulence [5].

On the other hand, in our case, the NS equations with constant physical properties are discretized on a staggered grid using a fourth-order symmetry-preserving discretization [6]. We propose to apply the same ideas to discretize the eddy-viscosity model (2) for LES (1). Since ν_e is not constant, the discretization of $\nabla \cdot (\nu_e (\nabla \boldsymbol{u})^T)$ needs to be addressed. This can be quite cumbersome especially for staggered formulations. The standard approach consists in discretizing the term $\nabla \cdot (\nu_e (\nabla \boldsymbol{u})^T)$ directly. However, this implies many *ad hoc* interpolations that tends to smear the eddy-viscosity, ν_e. This may (negatively?) influence the performance of eddy-viscosity especially near the walls. Instead, an alternative form has been proposed in [7]. Shortly, with the help of vector calculus it can be shown that $\nabla \cdot (\nu_e (\nabla \boldsymbol{u})^T) = \nabla(\nabla \cdot (\nu_e \boldsymbol{u})) - \nabla \cdot (\boldsymbol{u} \otimes \nabla \nu_e)$. Then, recalling that the flow is incompressible, the second term in the right-hand-side can be written as $\nabla \cdot (\boldsymbol{u} \otimes \nabla \nu_e) = (\boldsymbol{u} \cdot \nabla) \nabla \nu_e = \mathcal{C}(\boldsymbol{u}, \nabla \nu_e)$, *i.e.*

$$\nabla \cdot (\nu_e (\nabla \boldsymbol{u})^T) = \nabla(\nabla \cdot (\nu_e \boldsymbol{u})) - \mathcal{C}(\boldsymbol{u}, \nabla \nu_e). \tag{3}$$

In this way, consistent approximations of Eqs.(1)-(2) can be constructed without introducing new interpolation operators. To test the performance of these new turbulence models [5] in conjunction with the new discretization approach [7] is part of our research plans. In particular, we plan to test them for a turbulent square duct at $Re_\tau = 1200$.

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