

## EFFECTIVE CONSTITUTIVE EQUATION OF PLYWOOD BEAM MODEL

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The standard Timoshenko beam model assumes that cross-sections of the beam move as rigid bodies in deformation. This is a source of modelling error e.g. due to the warping effect. Therefore, shear and torsion correction factors are often introduced in the standard Timoshenko beam constitutive equation [1,2] to obtain an effective Timoshenko beam constitutive equation. The various methods for the correction factors are based on solving a beam problem under less restrictive assumptions than used in the standard model.

A computational homogenization method is suggested for finding the correction factors of effective constitutive equation. The idea is to compare the predictions by detailed and rough models on a RVE (Representative Volume Element) under periodicity assumption in the axial direction of beam. In the plywood beam application, the detailed model is based on the elastic properties of plies, stacking of plies, and the detailed geometry of cross-section. The rough model corresponds to the standard Timoshenko beam model.

The direct outcome of calculations is the effective constitutive equation of plywood beam model. In practice, finite element method is used to solve the six unknown functions associated with the detailed model on the domain of the cross-section. Discretized problem boils down to a linear equation system with six different right-hand sides.

In the illustrative numerical applications, shear correction factors, as predicted by computational homogenization, are compared to the ones in literature. In the plywood beam application, computational homogenization is used to find the effective constitutive equation as function of the ply properties, thickness, and number of plies and, thereby, obtain a picture about modelling error of the standard constitutive equation in plywood applications.

### REFERENCES

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