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INDUCTIVE VERIFICATION OF NUMERICAL METHODS FOR WELL-POSED PROBLEMS

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In recent years, verification and validation is a common framework for building confidence in the prediction by numerical simulation[1]. Validation addresses the appropriateness of the mathematical model in reproducing experimental data. Therefore validation provides evidences at a few sampling points associated with the experiments and it can be recognized as an inductive inference. On the other hand, verification addresses the correctness of numerical solution to a given model from the mathematical point of view. In a rigorous way, solution verification, which ensures mathematical correctness of the numerical method, should be performed by proof of convergence[2]. However complete proof of the numerical method is hardly obtained. In this work, we propose an inductive approach of solution verification.

In the verification process, the method of manufactured solution is used to evaluate discretization error by actual numerical calculations. In this method, artificial problems constructed from certain smooth solution fields are employed and source terms are generated from the given solution. Such problems are basically unrelated to the target problem of interest. For the proven numerical method, the correctness of the simulation code can be evaluated by manufactured solutions. However correctness of the unproven numerical method cannot be ensured by this approach. In this work, the method of manufactured solution is used in the inductive inference of correctness of the unproven numerical method. In our approach, the solution of the target problem is evaluated by several sampling points scattered near the solution of the target problem. Such sampling points are constructed from smooth numerical solutions, which can be derived from discretization scheme based on the sufficiently continuous bases such as spline or Fourier functions.

Furthermore, the numerical solution of the target problem is evaluated from the property of well-posed problem in which the solutions are continuous with respect to the source terms. Continuity of the solution of the target problem and manufactured solutions with respect to the source terms can be assessed by using the appropriate norm.

Consequently the numerical solution of the target problem can be demonstrated in an inductive way by using series of manufactured solutions.

Some examples show effectiveness of the present approach.

REFERENCES

[1] P.J. Roache. *Verification and Validation in Computational Science and Engineering.* Hermosa Pub., 1998.

[2] I. Babuska and J.T. Oden. Verification and validation in computational engineering and science: basic concepts. *Comput. Methods Appl. Mech. Engrg.* Vol. **193**, 4057–4066, 2004.