11th. World Congress on Computational Mechanics (WCCM XI)
5th. European Conference on Computational Mechanics (ECCM V)
6th. European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

A BALANCING PRECONDITIONER OF ITERATIVE DOMAIN DECOMPOSITION METHODS FOR MAGNETOSTATIC PROBLEMS

TAGAMI, Daisuke[1]

[1] Kyushu University; Fukuoka 819-0395, JAPAN; tagami@imi.kyushu-u.ac.jp

Key words: *balancing domain decomposition method, multigrid method, preconditioner, magnetostatic problem.*

1 Introduction

We have introduced an iterative domain decomposition method (DDM) to solve large scale computational models derived from electromagnetic field problems; see, for example, Kanayama, et al. [4] and Tagami [6]. In Tagami [6], we formulate the iterative DDM based on a mixed formulation of magnetostatic problems introduced in Kikuchi [2], [3], which enables us to prove unique solvability of the problems and convergency of the approximate solution.

However, it is difficult to figure out efficient implementations of iterative DDMs based on mixed formulations. Especially, in case of large scale computational models, the number of iterative steps is larger as the scale of computational models is larger. Therefore, it is required to introduce an efficient preconditioner of the iterative DDM steps.

In this paper, we introduce a balancing domain decomposition (BDD) method with a kind of multigrid strategy. The BDD method is originally proposed in Mandel [5], where the linear system is positive symmetric. Although the linear system in our case is indefinite, the BDD method enables us to keep the number of iterations of the iterative DDM even if the number of subdomains increases.

2 A Mixed Formulation of Magnetostatic Problems

Let Ω be a polyhedral domain with its boundary Γ and the outward unit normal $\boldsymbol{n}$. Let $\boldsymbol{u}$ denote the magnetic vector potential, $\boldsymbol{f}$ an excitation current density, and ν the magnetic reluctivity. Then, the magnetostatic equation with the magnetic vector potential and the Coulomb gauge condition is formulated as follows:

$$\begin{cases} \mathrm{rot}(\nu\,\mathrm{rot}\,\boldsymbol{u}) = \boldsymbol{f} & \text{in } \Omega, \quad (1a) \\ \mathrm{div}\,\boldsymbol{u} = 0 & \text{in } \Omega, \quad (1b) \\ \boldsymbol{u} \times \boldsymbol{n} = \boldsymbol{0} & \text{on } \Gamma. \quad (1c) \end{cases}$$

Now, following Kikuchi [2], a mixed weak formulation of magnetostatic problems with the Lagrange multiplier p is formulated. Moreover, as in Kikuchi [3], magnetic vector potential $\boldsymbol{u}$ is approximated by the Nedelec element of the first order and the Lagrange multiplier p is approximated by the conventional $P1$-element. Then, the resultant linear system is obtained as the finite element equation of magnetostatic problems with the mixed formulation.

3 A Balancing Domain Decomposition Method

As in Cowsar, et al. [1] and Mandel [5], a BDD method is formally applied into the finite element equation of magnetostatic problems with the mixed formulation, which is also regarded as a preconditioner of the iterative DDM in Tagami [6].

In the BDD methods, the null space of a coefficient matrix plays an important role. In case of magnetostatic problems with the mixed formulation, the null space consists of gradients of the $P1$-approximate Lagrange multipliers. Therefore, the dimension of the null space is equal to the number of the nodal points of finite element mesh. This is why the computational cost increases as the scale of computational models becomes lager. In order to settle this difficulty, a kind of multigrid strategy is introduced into the BDD method: set a coarse grid in each subdomain, and “1st order polynomials on the coarse grid” are adopted into solving the coarse grid problems in the BDD procedure. This strategy enable us to reduce computational costs when solving the coarse grid problems, and to keep the efficiency of the BDD method.

REFERENCES

[1] Cowsar, L.C., Mandel, J., and Wheeler, M.F., Balancing Domain Decomposition for Mixed Finite Elements, *Math. Comp.*, **64** (1995), 989–1015.

[2] Kikuchi, F., Mixed formulations for finite element analysis of magnetostatic and electrostatic problems, *Japan J. Appl. Math.*, **6** (1989), 209–221.

[3] Kikuchi, F., On a discrete compactness property for the Nedelec finite elements, *J. Fac. Sci. Univ. Tokyo, Sect. IA Math.*, **36** (1989), 479–490.

[4] Kanayama, H., Shioya, R., Tagami, D., and Matsumoto, S., 3-D Eddy Current Computation for a Transformer Tank, *COMPEL*, **21** (2002), 554–562.

[5] Mandel, J., Balancing Domain Decomposition, *Commun. Numer. Methods Engrg.*, **9** (1993), 233–241.

[6] Tagami, D., An Iterative Domain Decomposition Method with Mixed Formulations, in *COE Lect. Notes Series*, vol. 45, Kanayama, H. Ed., Kyushu University, 2013, 19–26.