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HIGHLY ACCURATE DISPERSION RELATION PRESERVING SCHEMES FOR INCOMPRESSIBLE FLOWS

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The results of a 3D DNS code based on explicit optimized DRP [1] are analyzed for moderate Reynolds number (*Re*=4200). The aim is to show the ability of these schemes to give accurate results in a small amount of time comparing to others accurate schemes as compact schemes. To this end, we consider a wall-bounded turbulent flow. Periodic boundaries conditions are applied in the streamwise and spanwise directions combined with Dirichlet boundaries conditions.

As others explicit schemes, DRP schemes perform the estimation using the values of the function $\boldsymbol{f}$ at the neighborhood points. The first derivative estimation $\boldsymbol{\delta f/\delta x}$ on a collocated grid is performed by a 7-points stencil here:

$$\frac{\delta f_i}{\delta x} = \sum_{j=1}^{7} a_j \frac{(f_{i+j} - f_{i-j})}{2jh}$$

The coefficients $\boldsymbol{a_j}$ of the scheme are determined by using a Tam & Webb like method [1].

A first measure of schemes accuracy can be obtained by applying them to a Fourier mode $f=e^{(ikx)}$. The accuracy decreases naturally with the wavenumber $\boldsymbol{k}$ and with the grid spacing $\boldsymbol{h}$. The figure 1 shows the accuracy given by the ratio $\frac{\delta f}{\delta x}/\frac{df}{dx}$ (where ***d/dx*** stands for exact derivative) as a function of $\boldsymbol{kh}$ product for a 7-points DRP scheme. The OUCS3 compact scheme [2] is chosen for this comparison because of its reasonable numerical cost and its good accuracy. The accuracy of the DRP scheme is clearly better than the one of the compact scheme OUCS3.

Finally, a DNS code configuration using DRP schemes has been compared with a configuration using classical second order explicit schemes and a configuration using compact schemes. The DRP configuration uses DRP schemes for streamwise and spanwise direction. For the wall normal direction, DRP schemes are only used for the interior points, whereas classical schemes are used for the boundaries points. The Compact Schemes (CS) configuration involves the optimized compact scheme OUCS3 for the first derivative at collocated node and classical compact schemes (6th order) for the rest of the estimations. The compact schemes used for the boundaries points involve a reduced stencil and their formal accuracy is reduced too (3rd order).

The results obtained for these configurations on a coarse grid (64x64x128) are compared to the Moser results [3] obtained by spectral methods on a finer grid (128x129x128). Figures 2, 3 and 4 show the velocity, *x*-vorticity and pressure RMS profile respectively in wall units for the three configurations. Despite the low resolution of the grid, the DRP and Compact configurations results are in a good agreement with the reference data. As expected, DRP and compact schemes give clearly better results than the second order explicit scheme.

It should be pointed out that, while being in excellent agreement with the reference data, the results of the DRP configurations have been obtained in a twice shorter time than them obtained from the compact configuration. DRP schemes seem to be a suitable option for the problem studied here which offers a good compromise between accuracy and numerical costs. Simulations at high Reynolds number (40 000) are currently performed on the Turing machine of IDRIS (National Computational Center). A first analysis of these results will be included in the final paper.

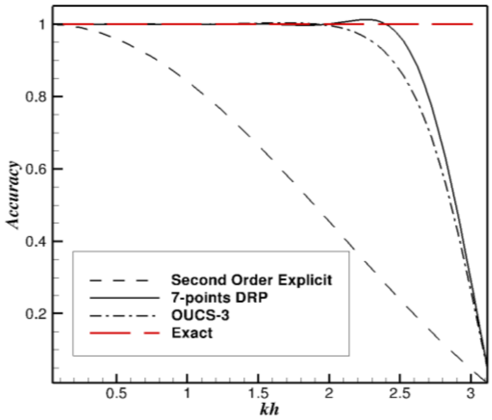

Fig.1. Accuracy of second order, DRP and compact schemes

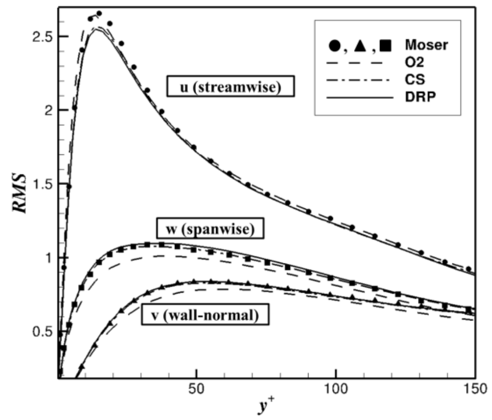

Fig.2. R.M.S of streamwise (u), wall-normal (v) and spanwise velocity components

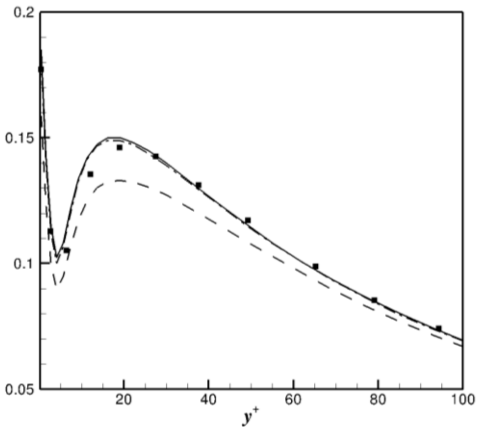

Fig.3. R.M.S of streamwise vorticity component (see Fig. 2 for caption)

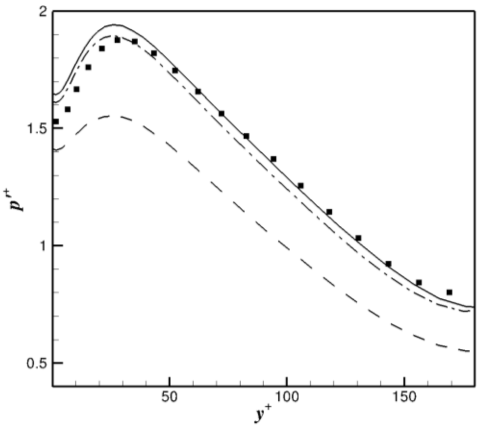

Fig.4. R.M.S of pressure

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