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HOMOCLINIC ORBITS AND THEIR RELEVANCE TO THE ONSET OF TRANSIENT TURBULENCE IN WALL FLOW

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Recent studies have shown that in transitional wall-bounded shear flows without linear instability of a laminar state the boundary between laminar and turbulent flows can be numerically identified in phase space as the edge of chaos [1]. A particularly interesting situation arises when the laminar-turbulent boundary is formed by the stable manifold of a traveling wave or periodic solution, which is then called a "simple edge state." Edge states have now been computed for flow in channels as well as pipes, with various numerical schemes, and there is some consensus that they are a robust feature of subcritical shear flows.

Here we study the two-dimensional unstable manifold of an edge-state periodic solution [2, 3] to the incompressible Navier–Stokes equations in plane Couette flow, using a novel computational algorithm [4, 5], and find that it contains two distinct orbits which return to the edge state along its stable manifold. We discover that these homoclinic orbits collide and disappear in a tangency bifurcation at lower Reynolds number. Away from this bifurcation, these orbits are transversal intersections of the stable and unstable manifolds. Thus, their presence generically implies the existence of an intricate tangle of these manifolds through the classical Smale–Birkhoff theorem. This, in turn, may explain the onset of transient turbulence.

We consider plane Couette flow in the minimal flow unit of dimensions $L_x \times 2h \times L_z$, where Re is based on half the velocity difference between the two walls, U, and half the wall separation, h. The streamwise and spanwise periods are $(L_x/h, L_z/h) = (1.755\pi, 1.2\pi)$ [2, 3]. The edge state in this computational domain is a time-periodic variation, which shows weak, meandering streamwise streaks [2, 3]. This gentle periodic orbit has a single unstable (real) Floquet multiplier and thus its unstable manifold has dimension two and its stable manifold has codimension one in phase space.

We track down a pair of two homoclinic orbits from/to the gentle periodic solution to the lowest Reynolds number as shown in Figure 1. The two homoclinic orbits are observed to collide at their tangency Reynolds number $Re = 241.3$ below which there is no homoclinic tangle. The Reynolds number of such a tangency bifurcation could be considered as the onset Reynolds number for a homoclinic tangle leading to transient turbulence, which should be significant for a theoretical interpretation of subcritical transition to turbulence in wall-bounded shear flow. In the presentation of this work, we shall discuss the identification of the onset of transient turbulence in terms of tangency of homoclinic orbits.

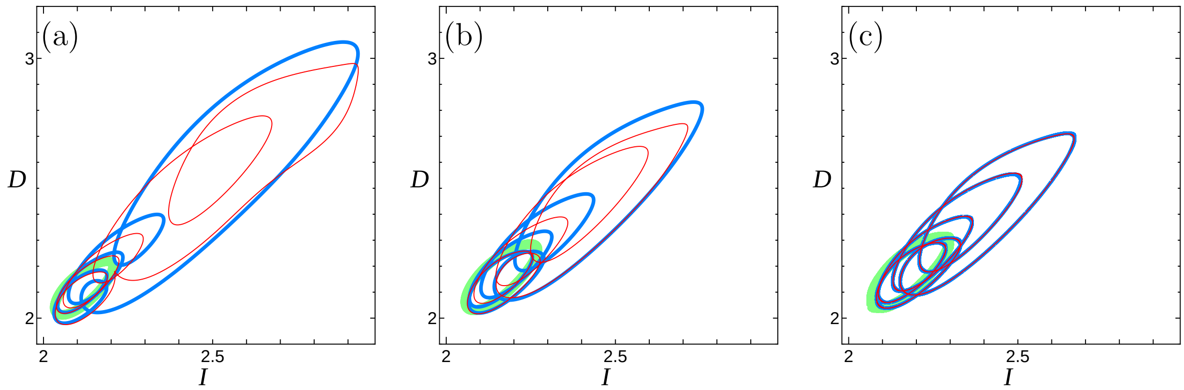

Figure 1. Two-dimensional projections of a pair of homoclinic orbits on energy input I – energy dissipation D plane. Blue curves denote the homoclinic orbit found by van Veen and Kawahara [5]. Red curves represent the newly discovered homoclinic orbit. Green loops are the lower-branch of the gentle periodic orbit. (a) $Re = 260$, (b) $Re = 245$, (c) $Re = 241.3$.

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