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ELASTIC FIELDS AND STRESS INTENSITY FACTORS OF CRACKS INTERACTING WITH INCLUSIONS

Kun Zhou* and Rongbing Wei

School of Mechanical and Aerospace Engineering, Nanyang Technological University,
50 Nanyang Avenue, Singapore, 639798
*kzhou@ntu.edu.sg

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The mechanical strength of a composite is greatly increased by reinforcing fibers or particles within its matrix material. However, these reinforcing components also cause stress concentration that often leads to fracture and plastic damage. For example, interfacial debonding between the matrix and the particles/fibers and cracking within the matrix are common damage modes of the composite. A proper prediction of the strength of a composite requires an accurate knowledge of its elastic fields, particularly when it contains cracks.

The reinforcing particles/fibers in the composite are essentially inhomogeneous inclusions with respect to the matrix. Thus, this paper develops a semi-analytic solution for multiple inhomogeneous inclusions of arbitrary shape and cracks in an isotropic infinite space. A novel method combining the equivalent inclusion method (EIM) [1] and the distributed dislocation technique (DDT) [2] is proposed. Each inhomogeneous inclusion is modeled as a homogenous inclusion with initial eigenstrain plus unknown equivalent eigenstrain using the EIM, and each crack of mixed modes I and II is modeled as a distribution of edge climb and glide dislocations with unknown densities. The solution is capable of taking into account the interactions among any number of inhomogeneous inclusions and cracks and thus provides an accurate description of their elastic fields and stress intensity factor (SIF) near the crack tip.

Consider an infinite space with elastic moduli $\mathbf{C}$ contains multiple cracks $\Gamma_\varphi\,(\varphi = 1, 2, \ldots, m)$ and inhomogeneous inclusions $\Omega_\psi\,(\psi = 1, 2, \ldots, n)$ with elastic moduli $\mathbf{C}^\psi$ under remote loading. Using EIM [1], the following governing equation can be established for any point within the inhomogeneous inclusions Ω_ψ:

$$(\mathbf{C}^\psi \mathbf{C}^{-1} - \mathbf{I})\boldsymbol{\sigma}^* + \mathbf{C}^\psi \boldsymbol{\varepsilon}^* = (\mathbf{I} - \mathbf{C}^\psi \mathbf{C}^{-1})(\boldsymbol{\sigma}^{\mathrm{P}} + \boldsymbol{\sigma}^{\mathrm{C}} + \boldsymbol{\sigma}^0). \quad (1)$$

In Eq. (1), $\mathbf{I}$ is a unit matrix, $\boldsymbol{\sigma}^*$ the eigenstress caused by the equivalent eigenstrains $\boldsymbol{\varepsilon}^*$ in all the homogeneous inclusions, $\boldsymbol{\sigma}^{\mathrm{P}}$ the eigenstress caused by the initial eigenstrains $\boldsymbol{\varepsilon}^{\mathrm{p}}$, $\boldsymbol{\sigma}^{\mathrm{C}}$ the stress due to all the cracks, and the stress $\boldsymbol{\sigma}^0$ due to remote loading. For any point along the cracks Γ_φ, the free surface traction conditions should be satisfied:

$$\boldsymbol{\sigma}^{\mathrm{P}} + \boldsymbol{\sigma}^* + \boldsymbol{\sigma}^0 + \boldsymbol{\sigma}^{\mathrm{C}} = 0\,. \quad (2)$$

Governing equations (1)-(2) are solved simultaneously by using the conjugate gradient method [3]. The SIF can be obtained by numerically approximate of dislocation densities [4].

Figure 1 shows the schematic of one crack and two square inhomogeneous inclusions Ω_1 and Ω_2 embedded in an infinite matrix under remote stresses $\sigma_y^0 = 1$ GPa . The matrix has

$E_M = 210$ GPa and $\nu_M = 0.28$; the two inclusions have $E_I = \lambda E_M$ and $\nu_I = 0.28$. The parameter $\lambda < 1$ represents a compliant inclusion; while $\lambda > 1$ represents a stiff inclusion. The inclusions Ω_1 and Ω_2 have the same side length a and are centered at $(-1.25a, 0)$ and $(1.25a, 0)$, respectively. The crack along the *x*-axis has the length of $2l = a$, and is centered at the (0, 0).

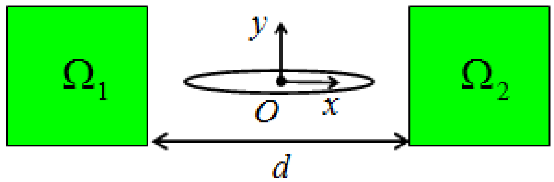

Fig. 1. One crack and two inclusions in an infinite matrix.

Figure 2a plots the contours of the von Mises stresses for the case of $\lambda = 1.6$ and shows strong stress concentrations around the crack tips and the inclusion corners. Figure 2b further plots the dependence of SIF at the crack tips on the Young's modulus of the inclusion. It shows that the SIF decreases as the inclusions becomes stiffer.

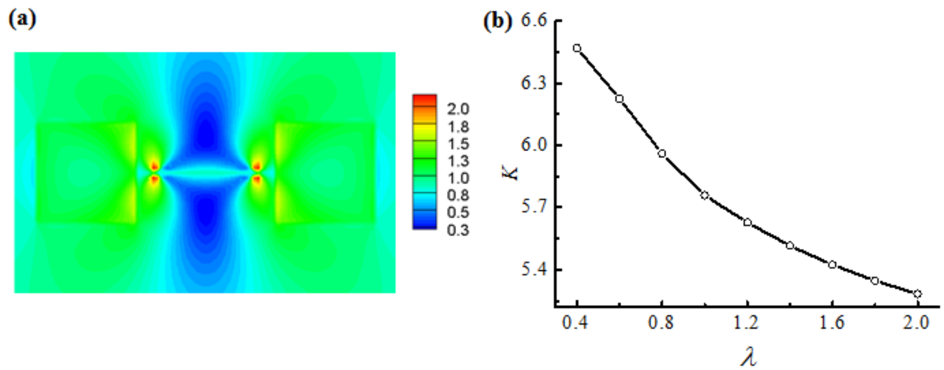

Fig. 2. (a) Contours of the normalized von Mises stress σ_v / σ_y^0 and (b) effect of the Young's modulus of the inclusions on the SIF ($\mathrm{MPa \cdot m^{1/2}}$) at the crack tips.

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