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PDE-CONSTRAINED OPTIMIZATION USING PROGRESSIVELY-CONSTRUCTED REDUCED-ORDER MODELS

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Optimization problems constrained by nonlinear Partial Differential Equations (PDE) arise in many engineering fields and contexts including inverse modeling, control, and shape optimization. Shape optimization problems in Computational Fluid Dynamics (CFD) are among the most computationally expensive PDE-constrained optimization problems, due to the large number of variables in the PDE state vector and potentially large number of shape parameters. As all optimization solution techniques are iterative in nature, the solution of a PDE-constrained optimization problem will likely involve many simulations of the high-dimensional PDE, depending on the optimization framework employed. For industry-scale problems, the solution of the PDE will be computationally expensive, implying the solution of the PDE-constrained optimization problem will be extremely expensive.

In this work, we replace the High-Dimensional Model (HDM), the CFD simulation in our case, with a Reduced-Order Model (ROM) in the optimization procedure. Ideally, the ROM has orders of magnitude fewer degrees of freedom than the HDM and can be run in a fraction of the time with a minor loss in accuracy (with respect to the HDM) [1]. The result is an approximation to the optimal solution at a small fraction of the cost required to solve the original PDE-constrained optimization problem.

The standard offline-online approach to reduced-order modeling breaks the computational effort into two distinct phases: a non-time-critical offline phase and a time-critical online phase. Respecting this breakdown in the context of optimization requires constructing a

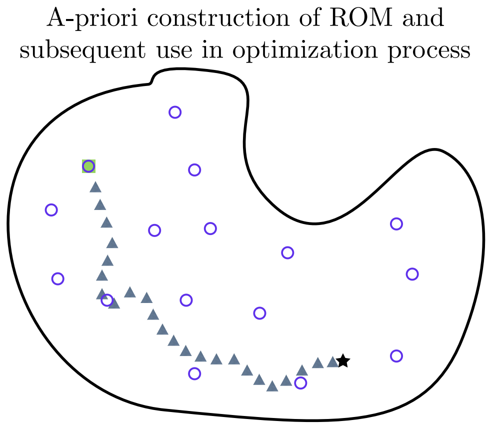

Figure 1: Illustration of described approaches to ROM-Constrained Optimization: ⋆ - optimal solution, △ - optimization iterate, ○ - HDM Sample, □ - initial guess for optimization

parametrically-robust ROM in the offline phase and using it as a surrogate for the HDM in the optimization problem. Construction of a ROM requires querying the HDM at some parameters and compressing the corresponding solution trajectories into a reduced basis. For highly nonlinear problems and problems with a large parameter space, constructing a ROM that will maintain reasonable accuracy throughout the parameter space is a difficult task that will require many HDM samples and a large reduced basis, which may impact potential speedups.

In this work, we break the offline-online approach to reduced-order modeling by constructing a ROM about the current iterate in the optimization procedure. The goal is to reduce the number of HDM samples by only requiring the ROM have parametric robustness *in the vicinity* of the trajectory of the optimization iterates. This is accomplished by merging the offline and online phases. A ROM is initially constructed at the optimization starting point and the corresponding ROM-constrained optimization problem is solved with an additional constraint on a residual-based error indicator of the reduced-order model. At the solution of this optimization sub-problem, the HDM is sampled and the solution trajectories are used to update the reduced basis. A schematic depicting the difference between the two optimization approaches is given in Figure 1. The algorithm repeats with the new reduced-order model until convergence is achieved. This project extends the work in [2] and provides a practical example from aerodynamic shape optimization.

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