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# Study of model order reduction based on POD for nonlinear dynamic response structural optimization

**Euiyoung Kim[1]*, Seongmin Chang[2] and Maenghyo Cho[3]**

[1*] Department of Mechanical & Aerospace Engineering, Seoul National University, 301-1415, Gwanak-ro 1, Gwanak-gu, Seoul, South Korea, takeme04@snu.ac.kr

[2] Department of Mechanical & Aerospace Engineering, Seoul National University, 301-1415, Gwanak-ro 1, Gwanak-gu, Seoul, South Korea, luxon@snu.ac.kr

[3] Department of Mechanical & Aerospace Engineering, Seoul National University, 301-1524, Gwanak-ro 1, Gwanak-gu, Seoul, South Korea, mhcho@snu.ac.kr

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Nonlinear dynamic response optimization requires a large number of nonlinear transient analyses as design variables change in the process of optimization. If a large-scale structural system is to be optimized, the computational cost for the transient analysis will be enormous.

Equivalent Static Loads (ESL) method has been proposed for the efficient way of nonlinear dynamic response optimization. The equivalent static loads are defined as the static loads which generate the same response fields in linear static analysis as those of nonlinear dynamic analysis at an arbitrary time[1]. ESL method provides efficient solution for nonlinear dynamic response optimization, since dynamic optimization can be treated as static optimization problem. However, ESLs are obtained by full order nonlinear transient analysis, which makes the method be lack of efficiency when it comes to the nonlinear transient analysis.

Reduced order model can be utilized to decrease the computational cost of the nonlinear dynamic optimization. Among many reduction methods, Proper Orthogonal Decomposition (POD)-based reduction scheme is popular for nonlinear structural dynamics problem. To construct POD-based reduced model, the response of full order nonlinear transient analysis is obtained at a few sampling points and saved in a snapshot matrix. POD modes are computed from this snapshot. In most cases, the sampled responses are obtained by performing full order response analysis during specific training period[2].

With relatively small number of POD modes, the response of full system can be represented well. This is why POD modes are used for model order reduction scheme. Let's say $\mathbf{V}$ has a few selected POD modes, then full order response $U$ and reduced system matrices can be represented by

$$U = \mathbf{V}U_R, \quad \mathbf{K}_R = \mathbf{V}^T\mathbf{K}\mathbf{V}, \quad \mathbf{M}_R = \mathbf{V}^T\mathbf{M}\mathbf{V}$$

where $\mathbf{K}$ is stiffness matrix and $\mathbf{M}$ is mass matrix( $R$ denotes reduced model). Then, the equation of motion of the finite element system is reduced as

$$\mathbf{M}_R \, {}^{t+\Delta t}\ddot{U}_R^{(k)} + {}^{t}\mathbf{K}_R \Delta U_R^{(k)} = {}^{t+\Delta t}\mathbf{F}_R - {}^{t+\Delta t}\mathbf{P}_R^{(k-1)}$$

where $\mathbf{F}$ and $\mathbf{P}$ are external force matrix and internal force matrix, respectively.

Since the design changes in optimization process alter the response of transient analysis, POD modes are obtained at each design iteration. This keeps computational cost high even with the reduced order model.

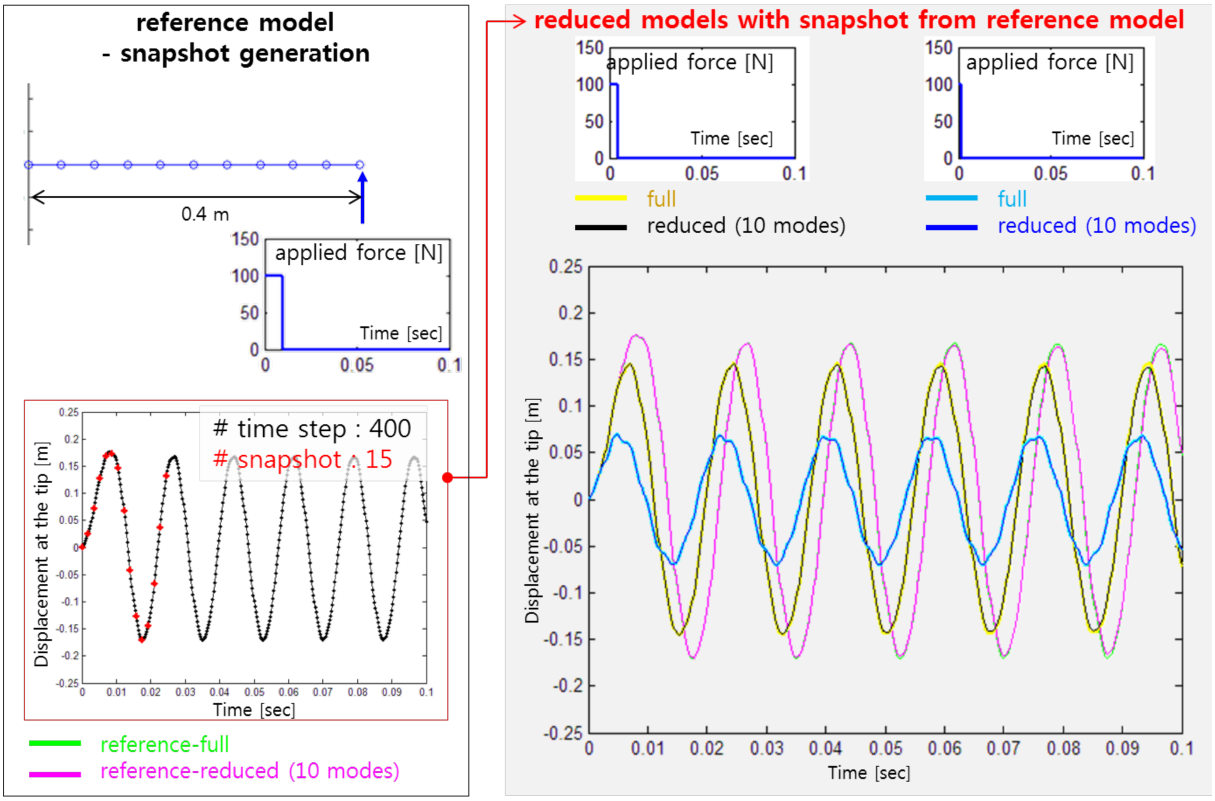


**Figure 1. Reuse of snapshot matrix**

This work focuses on the construction of POD-based reduced model for nonlinear dynamic response optimization using ESL method. The full order nonlinear response is sampled at the reduced number of sampling points. Moreover, POD modes are not re-computed at every design iteration.

The figure 1. simply shows the generation process of snapshot matrix (sampling points are marked as red) and the possibility of the reuse of the snapshot matrix for changed conditions. A structural nonlinear system of cantilever beam with dynamic force at the tip is reduced using POD modes. The same POD modes are used for the construction of reduced models with the altered conditions. Each reduced model yields accurate results.

The detailed sampling and updating strategies for the construction of POD-based reduced model will be added on this work. The proposed scheme increases the efficiency of nonlinear dynamic response optimization. The validation will be given by various structural design problems.

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