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FREESTREAM PRESERVATION ON A HIGH-ORDER CONSERVATIVE FR SCHEME

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Recently, the methods with high-order space accuracy are developed on unstructured meshes (discontinuous Galerkin (DG), spectral difference (SD), spectral volume (SV), and flux reconstruction (FR) schemes), which possess multiple degrees of freedom for achieving high-order accuracy in computational cells (elements) as shown in Fig. 1. In this study, much of the primary focus was given to the conservative FR scheme [5][3] that possesses an accuracy corresponding to that of the DG scheme.

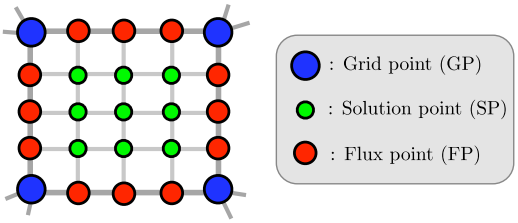

Figure 1: Multiple degrees of freedom in one cell in FR scheme

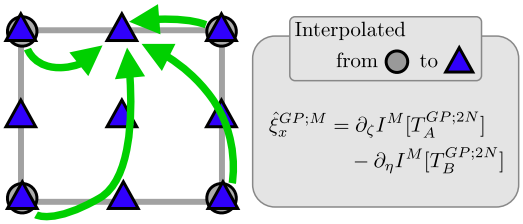

Figure 2: Schematic explanation for construction of conservative metrics

In the existing conventional high-order finite-difference schemes, the body-fitted coordinate system is often introduced for the computation of flows around complicated geometries. Because the boundary shape of each cell is analytically-defined by a high-order shape function in FR scheme, the computational grid, which should be prepared in advance, will be significantly reduced compared to high-order finite-difference schemes. However, the use of the body-fitted coordinate system and a high-order shape function often leads to violation of the freestream preservation [4] and integrated conservation [1][2] properties although the governing equation is expressed in a strong conservation form. This is caused by the violation of the discretized Leibnitz rule and commutativity of the multiple differencing operations due to the inappropriate computation of a coordinate-transformation

matrix, i.e., metrics. The versatile method has been proposed in the context of high-order finite-difference schemes [1][2], wherein the analytical expression of a metric is rewritten in its conservative form (hereafter, denoted as a conservative metric). The use of a conservative metric is expected to ensure the freestream preservation and integrated conservation properties also in FR scheme, but its implementation and validation are not straightforward. In this study, the following items are newly summarized:

(1) The conditions required for the freestream preservation and integrated conservation properties are shown with regard to conservative FR schemes.

(2) The new evaluation of conservative metrics which satisfies the freestream preservation is proposed and explained in detail while paying attention to the integrated conservation property and degree of polynomials for interpolation. One of the schematic explanations for construction of a conservative metric is shown in Fig. 2.

(3) Some verifications for the order of accuracy in numerical solutions are performed using conservative metrics. The isentropic vortex was computed on the wavy mesh, wherein the conservative metrics provide more accurate solution as shown in Fig. 3. The order of accuracy is also validated for the new conservative metric in Fig. 4.

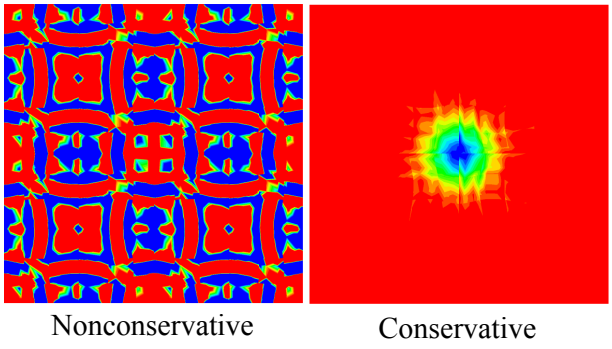

Figure 3: Vortex preservation on highly-distorted grid using nonconservative and newly-developed conservative metrics

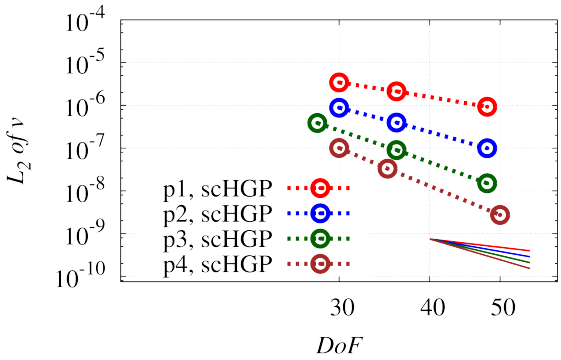

Figure 4: Order of accuracy for newly developed conservative metrics

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