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ANALYSIS OF AN ALGEBRAIC FLUX CORRECTION SCHEME

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Standard discretizations of convection-dominated problems, like central finite differences or the Galerkin finite element method, typically provide approximate solutions that are globally polluted with spurious oscillations. The reason is that solutions of convection-dominated problems usually possess layers, which cannot be resolved unless the given mesh is sufficiently fine in layer regions. A partial remedy is to apply so-called stabilized discretizations. There are many proposals of such discretizations, see the monograph [2] for an extensive review. Unfortunately, most of the methods fail to satisfy a discrete maximum principle. However, this property is particularly important in applications, where numerical results, e.g., with negative concentrations, will be considered to be worthless.

A very attractive choice seem to be algebraic flux correction schemes that satisfy a discrete maximum principle, but are usually nonlinear. However, applications often lead to nonlinear models, and then a nonlinear discretization of a linear equation in such a model seems not to be a severe disadvantage. Despite the attractiveness of algebraic flux correction schemes, there seems to be no rigorous numerical analysis for this class of methods. The main reason lies probably in their construction, which does not allow to apply the usual tools of the analysis of finite element discretizations. Unlike almost all other stabilized methods, which modify the bilinear form of the discrete problem in some way, algebraic flux correction schemes work on the algebraic level. They manipulate the matrix and the right-hand side of the algebraic system of equations.

In this work we study some properties of a nonlinear discrete problem that generalizes the algebraic flux correction method of TVD-type from [1] applied to the 1D steady-state convection–diffusion equation. The discrete operator of this scheme can be written as a nonlinear finite difference operator with an artificial diffusion vector whose components are bounded by a data-dependent constant $\tilde{\varepsilon}$. The discrete maximum principle for this operator can be proved for appropriately chosen values of $\tilde{\varepsilon}$. Different choices of $\tilde{\varepsilon}$, for which the discrete maximum principle is satisfied, will be studied numerically. A fixed point iteration is used for solving the nonlinear problem. While the linear subproblems in the fixed point iteration are proved to be well-posed, the nonlinear problem is not solvable in general, which will be demonstrated by several counterexamples. A modification of the scheme will be proposed for which the existence of a solution and a weak variant of the discrete maximum principle can be proved. To the authors' best knowledge, the results concerning the solvability of the linear subproblems and the nonlinear problem are the first results of this kind for algebraic flux correction schemes. In addition, the present work represents a basis for analyzing algebraic flux correction schemes applied to multi-dimensional problems.

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