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Hierarchical Model (HiMod) Reduction for Incompressible Fluid Dynamics in Rigid and Deformable Pipes

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Networks perfused by fluids are found in several engineering applications ranging from environmental problems (rivers), oil distribution, internal combustion engines and hemodynamics. In these problems the local domain of interest features a prevalent direction affecting the dynamics of the fluid of interest. One may take advantage of this, by simulating the problem only with respect to the local axial main direction. Euler equations are a well-known example for both gas dynamics and blood in compliant vessels. More generally, the dynamics can be regarded as the combination of the prevalent (axial) and the transverse components. In [1,2,3] we have proposed a numerical approach where starting from a 1D representation of the problem, the transverse components are progressively added by a spectral discretization, resulting in an overall "psychologically" 1D enriched hierarchical model (HiMod). The main idea is sketched in Figure 1. The rationale behind this approach is that a relatively small number of modes is expected to be necessary to get a reasonably accurate numerical solution. In addition, the accuracy of the reduced model can be modulated by selecting a proper number of transverse modes along the mainstream.
This approach is somehow a compromise between a purely 1D model and the *geometrical multiscale* approach advocated in [5] to couple dimensionally heterogeneous solvers.

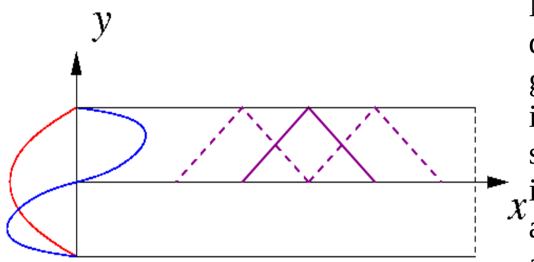

Figure 1 Schematic representation of the HiMod approach. Finite element discretization along the axis of the pipe is coupled to a spectral discretization for the transverse component.

Moving from preliminary studies for advection-diffusion problems, in this talk we address the generalization of this method to the 3D incompressible Navier-Stokes equations and fluid structure interaction problems. In particular, interesting issues addressed in this talk are:
a) the automatic choice of the modal discretization along the pipe based on a goal oriented a posteriori error estimation;
b) the generation of a non trivial spectral basis which automatically includes the boundary conditions assigned on the lateral boundary;
c) the comparison with the more established geometrical multiscale model (Fig. 2 and [4]);
d) the extension of the HiMod approach to 3D problems;
e) the extension of the HiMod approach to deformable domains;
f) the extension of the solution to the incompressible Navier-Stokes equations.
Preliminary results show that the HiMod approach potentially can reduce the problem to a system of coupled 1D equations still preserving a good accuracy for the transverse dynamics (Figs. 2 and 3). However, several issues need to be addressed, including the identification of

inf-sup compatible pairs of HiMod discretization space for the incompressible Navier-Stokes equations, the associate convergence analysis of the solution and the set up of effective algebraic solvers that maximize the advantage from the 1D-like structure of the associated linear systems.

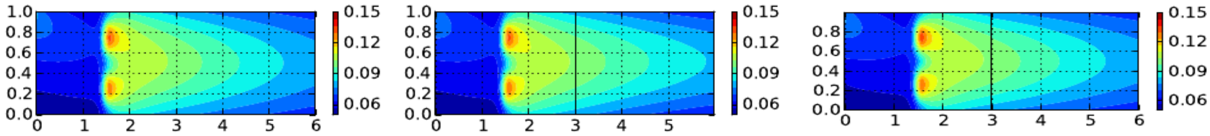

Fig.2 - Test case where a transverse dynamics is triggered by a forcing term acting in two circles of the leftmost half of the domain. Left: Full solution; Center: Geometrical multiscale approach, where the right half of the domain is solved by a 1D (transverse-averaged) model; Right: HiMod solution with $m_{left}=5$ and $m_{right}=1$. The agreement is excellent.

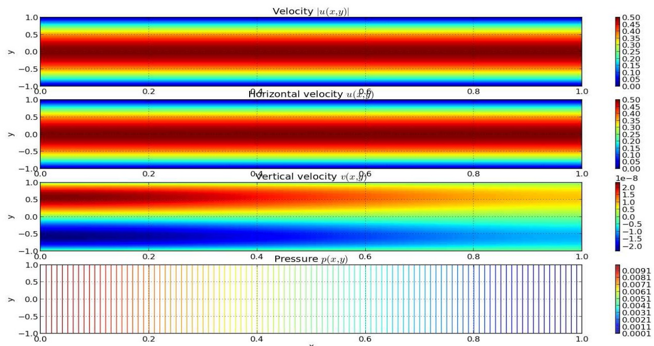

Figure 3 – HiMod for the incompressible Stokes equations. From top to bottom: velocity magnitude, horizontal velocity, vertical velocity (notice that the scale is multiplied by 10^{-8}), pressure contourlines.

REFERENCES

[1] Perotto, Simona, Alexandre Ern, and Alessandro Veneziani. "Hierarchical local model reduction for elliptic problems: a domain decomposition approach." *Multiscale Modeling & Simulation* 8, no. 4 (2010): 1102-1127.

[2] Perotto, S. "Hierarchical model (Hi-Mod) reduction in non-rectilinear domains. Accepted for the publication in Proceedings of the 21st International Conference on Domain Decomposition Methods." (2013).

[3] Perotto, Simona and Veneziani, Alessandro, "Coupled Model and Grid Adaptivity in Hierarchical Reduction of Elliptic Problems", *Journal of Scientific Computing*, On Line Publication ahead of Printing, available at http://link.springer.com/article/10.1007%2Fs10915-013-9804-y# , DOI: 10.1007/s10915-013-9804-y (2013)

[4] Aletti, Matteo, Andrea Bortolossi, Simona Perotto and Alessandro Veneziani, "One-dimensional surrogate models for advection-diffusion problems", Tech Report TR-2013-011 Dept. Math&CS Emory University, available at http://www.mathcs.emory.edu/publications.php (2013)

[5] Formaggia, Luca, Fabio Nobile, Alfio Quarteroni, and Alessandro Veneziani. "Multiscale modelling of the circulatory system: a preliminary analysis." *Computing and visualization in science* 2, no. 2-3 (1999): 75-83.