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ISOGEOMETRIC LARGE DEFORMATION 3D TIMOSHENKO BEAM

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An isogeometric beam formulation is derived from a 3D continuum, where large-deformation kinematics (Green-Lagrange strain tensor and second Piola-Kirchhoff stress tensor) and the St. Venant–Kirchhoff constitutive law are assumed. The beam's cross-sections are assumed to be symmetric and planar, reducing the modelling to a curve in 3D space. The initial configuration may be curved and twisted. The formulation accounts for membrane, bending, transverse shear and torsional effects, employing six degrees of freedom. The variational equations of equilibrium are derived from the equilibrium of virtual work, where integration over the beam volume is performed in the undeformed configuration. We use NURBS of order $p \geq 2$ as basis functions, resulting in C^1 or greater continuity across element boundaries.

The beam is represented in a local curvilinear coordinate system where ξ_1 is the parametric variable defining the beam's middle curve, and ξ_2 and ξ_3 parametrize the cross-section. $\mathbf{X}(\xi_1)$ and $\mathbf{x}(\xi_1)$ denote the beam's middle curve coordinates in the reference and deformed configuration, respectively. The displacement of the middle curve is given by $\mathbf{d}(\xi_1) = \mathbf{x}(\xi_1) - \mathbf{X}(\xi_1)$. The local curvilinear coordinate system consisting of the tangent $(\mathbf{T}, \mathbf{t})$, binormal $(\mathbf{B}, \mathbf{b})$ and normal $(\mathbf{N}, \mathbf{n})$ vectors is illustrated in Figure 1. The tangent vectors are calculated directly from the beam's geometry, while the normal and binormal vectors are calculated by applying a rotational operator, i.e. Rodriguez' formula, to the fixed triad $\{\mathbf{T}_0, \mathbf{B}_0, \mathbf{N}_0\}$ in $\xi_1 = 0$. The 3D beam geometry is defined by

$$\mathbf{X}^{3D}(\xi_1, \xi_2, \xi_3) = \mathbf{X}(\xi_1) + \xi_2 \mathbf{B}(\xi_1) + \xi_3 \mathbf{N}(\xi_1), \tag{1}$$

$$\mathbf{x}^{3D}(\xi_1, \xi_2, \xi_3) = \mathbf{x}(\xi_1) \ + \xi_2 \mathbf{b}(\xi_1) + \xi_3 \mathbf{n}(\xi_1). \tag{2}$$

The authors showed a similar beam formulation without transverse shear and torsion and with only displacement degrees of freedom in [1].

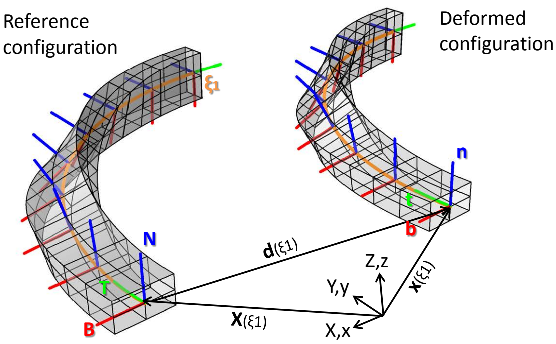
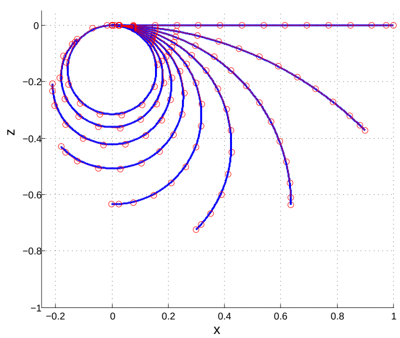


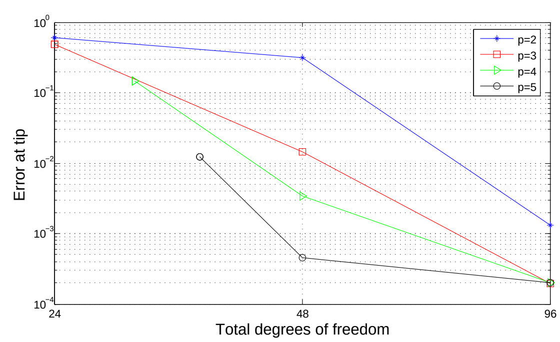
Figure 1: Sketch of the 3D beam in both the reference and the current configuration, showing the beam's middle curve and the local coordinate system consisting of the tangent $(\mathbf{T}, \mathbf{t})$, binormal $(\mathbf{B}, \mathbf{b})$ and normal $(\mathbf{N}, \mathbf{n})$ vectors.

Numerical example: Cantilever beam subjected to a constant tip moment

A cantilever beam of length $L = 1$ is subjected to a constant tip moment of $M_y = 2\pi$ in eight equal load steps. The magnitude of the moment is chosen such that the tip rotates 360° about the y-axis, deforming an initially straight beam oriented along the x-axis into a circle. The bending stiffness is set to $EI = 1.0$ and the membrane stiffness to $EA = 1.0e4$. The polynomial order of the NURBS-curve range from $p = 2$ to $p = 5$. The results for $p = 3$ with 16 global basis functions, corresponding to 13 elements or 96 total d.o.f., are presented in Figure 2a). The deformed configuration of the beam is almost coincident with the exact circle. Figure 2b) shows the convergence for the polynomial orders.



(a) Beam displacements. The red circles are the control points.



(b) Convergence plot for tip z-displacements.

Figure 2: Cantilever beam subjected to a constant tip moment.

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