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NUMERICAL SOLUTION OF HIGH DIMENSIONAL FOKKER-PLANCK EQUATIONS IN NONLINEAR STOCHASTIC DYNAMICS

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Uncertainty quantification and its propagation in structural dynamics is usually a sophisticated task, which demands elaborate stochastic modelling and entails huge computation times ([3]). If nonlinearities are present, the computational demand is even higher. Furthermore, most numerical stochastic models provide discrete samples as a solution, whereas rather probability density distributions would be the quantities of interest. In order to obtain probability density distributions from sample solutions, nested combinations of stochastic models are required. These make error estimation of the solution probability distribution very complicated. However, if the stochastic model is restricted to a Markov process, the behaviour of nonlinear oscillators under stochastic excitation can be modelled by the Fokker-Planck equation ([2]). In doing so, a transport equation for the probability density function itself is obtained and must be numerically solved. Error estimation for the solution of this deterministic equation is a standard task. Nevertheless, since the Fokker-Planck equation is of advection-diffusion type, it poses a challenge regarding stability of the numerical solution, especially if the hyperbolic part is dominant. Furthermore, the Fokker-Planck equation is defined on a high-dimensional space, corresponding to the number of degrees of freedom of the underlying stochastic dynamical system.

In order to accomplish the solution of the high-dimensional transient Fokker-Planck equation, a stable explicit numerical scheme, which allows for element-wise decoupling, has to be established. Discontinuous Galerkin methods, as a higher order extension to finite volume methods are very suitable for the solution of hyperbolic equations (see e.g. [1]). The discontinuous finite elements are intrinsically defined element-wise, whilst the coupling to neighbouring elements is established by inter-element fluxes. This allows for fully element-wise decoupling of the solution which provides the possibility for element-wise

parallelisation.

The Discontinuous Galerkin method has been implemented for arbitrary high-dimensional Fokker-Planck equations with arbitrary high order polynomial approximations in each element. Efficient concepts for mesh generation in arbitrary dimensions and storage of high-dimensional node-to-element and neighbourhood relations have been developed. Numerical integration over high-dimensional domains have been established, as well as the computation of surface integrals and surface normals for the numerical inter-element fluxes. With this explicit method, the solution of the high-dimensional system of equations is obtained with an efficient use of computational resources.

Adaptive mesh refinement techniques should also be taken into account, since hanging nodes fit well into the framework of the discontinuous Galerkin method. H-refinement and p-refinement are likewise appropriate and should be implemented.

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