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FINITE ELEMENT ERROR ESTIMATION FOR BOUNDARY VALUE AND EIGENVALUE PROBLEMS ASSOCIATED WITH A SCHRÖDINGER OPERATOR

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Error estimation and adaptive approximation are essential components of high-performance finite element computations. In this talk we consider both boundary value problems (BVPs) and eigenvalue problems (EPs) for a Schrödinger operator with inverse-square potential, $-\Delta + c^2 r^{-2}$. The inverse-square potential $c^2 r^{-2}$ not only gives rise to new sources of singularities in the solution of BVPs and EPs when $c > 0$ (the case $c = 0$ is the familiar Laplacian), but also requires a different approach to both analysis and numerical analysis. We will discuss an effective means of estimating errors in finite element discretizations using hierarchical bases, and demonstrate how one might use them to adaptively improve approximations in an efficient way. Key results include equivalence of error and error estimate in L^2, asymptotic exactness in energy, and convergence of the Hessian in $W^{2,1}$. Analysis is carried out on families of triangulations which are geometrically graded based on *a priori* knowledge of worst case singularities (which possible for the model problem), and empirical comparisons are made between this *a priori* approach and families of triangulations which are adaptively refined using local error indicators.

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