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Coupling Asymptotic Numerical Method and Proper Generalized Decomposition in bifurcation problems

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To deal with transient problems, the PGD (Proper Generalized Decomposition) [1] method has proved to be efficient thanks to its ability to handle the time coordinate in a non-incremental fashion. The main idea of the PGD is to look for a solution as the sum of products of functions of each variable, which reduces drastically computational costs. But one difficulty of the PGD lies on the resolution of non-linear problem which is usually done by linearization.

One widely used method for solving non-linear problem is Asymptotic Numerical Method (ANM) [2]. By introducing an expansion to the equilibrium equation, ANM consists in transforming the non-linear problem into a sequence of linear problems in recurrence. However, one of ANM drawback is its poor solution in case of transient problems. In a recent paper [3], we introduce a coupling of PGD method and ANM to solve transient non-linear problems. The numerical example is illustrated by a 1D heat equation:

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + T^2 + f_T(x,t) \qquad (1)$$

By using ANM, let us consider a loading parameter ϕ of the non-linear term:

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \phi \times T^2 + f_T(x,t) \qquad (2)$$

where T and ϕ are supposed to be in the polynomial form of parameter a :

$$\begin{aligned} \phi &= \phi_0 + a\phi_1 + a^2\phi_2 + \ldots a^p\phi_p \\ T &= T_0 + aT_1 + a^2T_2 + \ldots + a^pT_p \end{aligned} \qquad (3)$$

In the procedure of ANM, a linear problem is solved in each order of parameter a. The resolution of these transient linear problems is done by PGD method. At each order, the same differential operator is used, leading to an efficient way to compute the non linear solution.

The construction of bifurcation diagrams in complex bifurcation problems involving dissipative systems is usually time consuming. Terragni and Vega [4] have recently proposed to use POD based Reduced Order Models to be more computationally efficient. As the ANM is an efficient method to compute bifurcation branches [5], we here extend our alternative approach, ANM-PGD, to solve a simple bifurcation problem for a Fisher like equation as suggested in [4]. The equation is recalled here:

$$\frac{\partial u}{\partial t} = 0.1\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + 1 + \mu u - u^3 \quad (4)$$

with homogeneous boundary conditions at x equals 0 and 1 and where μ is the bifurcation parameter.

The results are discussed in terms of number of enrichments and time saving and compared to those obtained by Terragani et al. [4]. Furthermore the ability of the PGD to treat parameters such as the bifurcation parameter as additional coordinates of the problem is explored.

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