11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

COARSE–GRAINING APPROACHES FOR PARTICULATE COMPOSITES AS MICROPOLAR CONTINUA

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Key words: *Random composites, Cosserat continua, statistical homogenization, Representative Volume Element.*

Several composite materials, extensively adopted in many engineering fields, are characterized by particulate random microstructures. Examples are polymer, ceramic, metal matrix composites or also concrete, granular materials and porous rocks. A key issue in mechanics of materials characterized by microstructural randomness is that the classical concept of the Representative Volume Element (RVE), well established in periodicity based homogenization techniques since many years, loses its validity [1]. In the last few years, various procedures based on the solution of specific Boundary Value Problems (BVPs) have been proposed to perform classical homogenization for non–periodic assemblies [2, 3]. In order to account for the effects of the microstructural size, heterogeneous non–periodic materials have been also studied by extending the homogenization schemes to gradient–enhanced continua, although applied to a single fixed mesoscale [4]. Stochastic approaches based on finite–size scaling homogenization, have proved to be among the most effective for individuating the RVE size and the overall constitutive moduli in the linear elastic and thermoelastic, as well as in the non–linear and non–elastic, frame [5, 1].

In this work we adopt the statistically–based scale–dependent homogenization procedure developed in [6], which enables to simulate the actual microstructure of a simplified two–phase random media and to estimate the constitutive moduli of energy equivalent micropolar continua. This procedure uses finite–size scaling of Statistical Volume Elements (SVEs) and approaches the so–called Representative Volume Element (RVE) through two hierarchies of constitutive bounds, respectively stemming from the numerical solution of Dirichlet and Neumann non-classical boundary value problems, set up on mesoscale material cells. For defining these problems we use a generalized macro-homogeneity (Hill–Mandel type) condition, which accounts for non–symmetric stress and strain as well as couple–stress and curvature tensors. In particular, for a two–dimensional elastic medium made of a base matrix and a random distribution of disk–shaped inclusions of given density, two hierarchies of constitutive bounds are obtained by considering mesoscale test–windows of different sizes supposed placed anywhere in a random material domain. Under the hypotheses of statistical homogeneity and mean–ergodicity of the medium: the convergence trend of the bounds is detected as function of the SVE size; the RVE size is attained on the basis of a statistical criterion; the average homogenized, classical and micropolar, elastic moduli are estimated. The results of the simulations performed point out the importance of taking into account the spatial randomness of the medium, and in particular the presence of inclusions that intersect the edges of the test windows. The worthiness of accounting the additional stress and strain measures of the Cosserat continuum for describing random materials, widely investigated for anisotropic periodic media [7, 8], is also discussed.

Acknowledgments. This research has been partially supported by MIUR, Sapienza research grant 2011; Prin 2010-11, and by the NSF under grant CMMI–1030940.

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