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OPTIMIZATION OF A RANDOM CAUCHY PROBLEM IN LINEAR ELASTICITY

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The non-destructive inspection of material enables to detect damages, inclusions, and material complex properties in industrial structures. The solution is obtained from solving one inverse problem and using over-specified data measured on accessible boundaries. One approach called data completion allows to use only the available data on one part of the boundary and to reconstruct the missing data on inaccessible boundaries (e.g. internal boundaries).

However, due to complex phenomena to take into account, these models may be not sufficient to represent the reality: it is then necessary to randomize some uncertain data of the problem (on accessible or inaccessible boundaries) and then build one stochastic model.

The data completion problem also known as the Cauchy problem is one ill-posed problem where the lacking data do not depend continuously on the measured data. Various ways for solving such problems can be found in the literature, in particular Tikhonov and Arsenin [1], Bui [2], Kozlov et al [3], and Marin and Lesnic [4]. Here, we choose to use a new computational algorithm for the reconstruction of lacking data, which is based on the minimisation of energy functional: the Constitutive Relation Error (CRE) by using the data on accessible boundaries. This approach was used for the Laplace problem to that of linear elasticity [5-8].

The CRE is well known by its efficiency for classical identification problems [9] and was extended in the stochastic domain for modelling uncertain properties in industrial structures [10-12]. The stochastic method used was the Polynomial Chaos Expansion (PCE) [13] ; this method was applied in various domains with success. Here we propose to extend the works on the Cauchy problems [5-8] in the stochastic domain by using the PCE.

The problem to solve is a two-dimensional static ill-posed problem consisting in one structure with over-specified data measured on one accessible boundary. The material properties are considered as uncertain and then modelled as random quantities.

The stochastic problem is expanded on the chaos basis. Its minimization gives the random missing data (Neumann and Dirichlet). The functional is defined as the stochastic average energy error between the random solutions of two well-posed problems. The first has measured Dirichlet boundary data on the accessible part of the boundary and the unknown Neumann boundary data on the other part. The second has measured Neumann boundary data on the accessible part of the boundary and unknown Dirichlet boundary data on the other part. The minimisation of the average error gives the random solutions through their mean and their variance. Besides, to accelerate the convergence of such large-scale problems, the gradients of the energy functional are calculated using the stochastic adjoint problems.

The method is both very efficient with respect to the accuracy of the solution and regarding the amount of computation needed. The formulation is very general and can be used with heterogeneous materials and other physical phenomena.

The results are in good agreement with those obtained by the exact solution, which is computed by the Monte Carlo method. The minimisation is done one the stochastic error associated to each sample. The final solutions are given by their mean and their variance that are calculated a posteriori.

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