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# RETINAL BLOOD FLOW CHANGES AND VASCULAR PARAMETERS AND STRUCTURE

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The geometry and function of the retinal vascular supply to the eye in patients with ocular diseases, such as glaucoma, diabetic retinopathy or myopia, changes compared to the healthy eye. The eyes of patients with glaucoma, for instance, show a higher cup to disc ratio, increased retinal nerve fiber layer defects, decreased retinal arteriolar and venular tortuosity, narrower retinal branching angles and decreased retinal vascular fractal dimensions [1]. These changes in vascular anatomy could be related to the reduction in ocular blood flow velocities described in the literature.

Blood flow in the retina is a multiscale phenomena; on the largest scale, blood flows through arterioles and venules, with scale changes down to the smallest scale where single red blood cells move through the capillaries. To integrate these different scales we use continuum models, network models, and fractal models.

Continuum models are used where flow properties (pressure, velocity) can be measured. Following [2, 3] we model the blood flow in the retina as the flow of a fluid (blood) through a porous medium (retina tissue) with multiple layers representing the arteriolar, capillary and venous scales. Network models keep track of individual blood vessels. Vasculature conforms to a variational principle for optimal transport of the blood to minimize the energy to overcome shear stress of laminar (Poiseuille) flow in cylindrical pipes and the metabolic energy required to maintain blood volume and vessel tissue [4]. Currently only the first few levels of the vascular tree can be more or less accurately extracted from retinal images. Some assumptions need to be made in order to model the deeper levels of the tree [5]. Fractal models are used to simplify the complexity in network models [6].

To solve the continuum model we use finite elements and discrete exterior calculus (DEC) [7, 8]. Using exterior calculus the vector calculus operators (gradient, divergence, curl) can be expressed in a coordinate free notation in terms of the exterior derivative, the wedge product and the Hodge star operator. DEC uses the concept of a dual mesh to discretize these operators and reduces partial differential equations to matrix equations [9]. These methods are advantageous for problems on curved embedded manifolds and they aim to preserve the geometrical structure (flow, energy, symplecticity) of the original equations.

We apply a combination of multiscale and statistical methods to investigate the relation between ocular blood flow and vascular supply parameters, such as locally varying material properties, eye shape, and vascular anatomy. This is clinically important in order to understand the relation between eye blinding diseases and ocular blood circulation.

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