11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

NONLINEAR MANIFOLD FOR MODEL ORDER REDUCTION OF GEOMETRICALLY NONLINEAR STRUCTURAL DYNAMICS

P. Tiso[1], J. Rutzmoser[2] and D.J. Rixen [2]

[1] Faculty of Mechanical, Maritime and Materials Engineering
Delft University of Technology
Mekelweg 2, 2628CD, Delft, the Netherlands
email: p.tiso@tudelft.nl
[2]Chair of Applied Mechanics
Munich Technical University
Boltzmannstr. 15, 85748 Garching, Duitsland
email: {johannes.rutzmoser}{rixen}@tum.de

Key words: *Geometric Nonlinearities, Structural Dynamics, Model Order Reduction, Modal Derivatives, Nonlinear Manifolds.*

Finite element geometric nonlinear transient analysis of structures is a computationally demanding task. The nonlinear internal forces term needs to be evaluated at every time step during the time integration. If a consistent Newton-Rapshon algorithm is used, the evaluation of the jacobian needs to be performed at every residual iteration within a time step.

To alleviate the computational burden, one can seek for the solution in a much reduced subspace spanned by a basis containing the essential modes. It has been shown that a basis comprising relevant vibration modes together with (some) of their modal derivatives is very effective in yielding an accurate solution with a limited number of modal degrees of freedom [1].

However, the number of modal derivatives scales quadratically with the size of the corresponding set of vibration modes. It is possible to select the most relevant of these derivatives by examining the spectral and spatial convergence of the linearized dynamic problem and therefore reduce the size of the reduction basis [2]. However, the construction of the modal derivatives suggests the direct use of a nonlinear manifold to reduce the equations of motion. In this way, the number of modal coordinates is equal to the number of vibration modes used to generate the nonlinear expansion.

We present in this contribution a reduction method based on the projection of the discretized governing equations on a nonlinear manifold constructed as a second order expansion of the generalized displacements $\mathbf{u}(t)$ in the direction of the chosen M vibration modes $\mathbf{\Phi}_i,\ i = 1,\ldots,M$:

$$\mathbf{u}(t) = \mathbf{\Phi}_i q_i(t) + \mathbf{\Theta}_{ij} q_i(t) q_j(t). \tag{1}$$

The quadratic terms $\mathbf{\Theta}_{ij}$ are formed by the derivatives of the vibration modes with respect of the modal amplitudes, as:

$$\mathbf{\Theta}_{ij} = \frac{1}{2}\left(\frac{\partial \mathbf{\Phi}_i}{\partial q_j} + \frac{\partial \mathbf{\Phi}_j}{\partial q_i}\right) \tag{2}$$

Preliminary results are shown in Figure 1. The nonlinear manifold approximation and the projection on the tangent subspace introduce configuration dependent and convective terms in the inertial forces. The structure of these new terms and their impact on the time integration will be discussed.

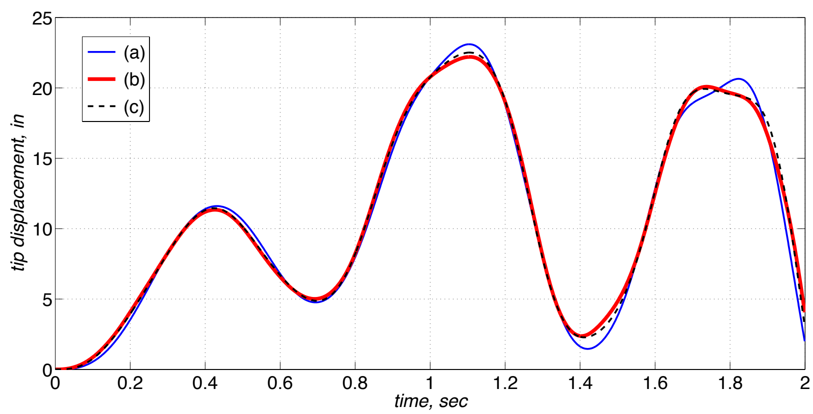

Figure 1: Tip vertical displacement of a cantilever beam subjected to harmonic tip vertical load. (a): nonlinear manifold as in eqn. (1) (3 vibration modes); (b): full solution; (c): linear manifold (3 vibration modes and 6 modal derivatives). The proposed approach is able to yield fairly good results with a remarkably reduced number of degrees of freedom (3 vs 9).

Different methods of calculating the modal derivatives terms will also be presented, and the trade between accuracy and computational effort will be discussed on beam and shell structural examples.

REFERENCES

[1] Sergio R. Idelsohn, Alberto Cardona, A load-dependent basis for reduced nonlinear structural dynamics, *Computers & Structures*, Vol. 20, Issues 13, 203-20, 1985.

[2] Paolo Tiso, Optimal second order reduction basis selection for nonlinear transient analysis. *Modal Analysis Topics*, Vol. 3, 27-39, 2011, Springer.