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A VORTICITY, ENSTROPHY, MASS AND ENERGY CONSERVING DISCRETIZATION FOR INCOMPRESSIBLE EULER EQUATIONS

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The modelling of hurricanes or ocean eddies are examples where incompressible Euler equations play a fundamental role. These equations are given by the following set of PDEs for $(\vec{u}, p)$ for a domain Ω,

$$\frac{\partial \vec{u}}{\partial t} + (\nabla \cdot \vec{u})\,\vec{u} + \nabla p = 0, \quad \nabla \cdot \vec{u} = 0, \quad \text{and} \quad \vec{u} \cdot \vec{n} = 0 \text{ on } \partial\Omega, \tag{1}$$

This system models the flow of an inviscid, incompressible fluid with constant density. It is widely known that under the hood this system is rich in geometric structures [1]. This richness cannot be ignored while discretizing the equations because of its connection with secundary conservation laws, i.e. enstrophy and kinetic energy. It is therefore important that the numerical scheme is compatible with these structures, that is to mimic them. The connection between physics and geometry is not unique to the motion of a perfect fluid but it can be found in any physical system [2, 3]. An extended review for the importance of preserving physical invariants for fluid flows can be found in [4]. A review of the framework used on this work can be found in [5].

Vorticity plays a fundamental role in fluid dynamics and it is possible to describe its dynamics of the two-dimensional incompressible Euler equations as,

$$\frac{\partial \vec{\omega}}{\partial t} + \nabla \times (\vec{\omega} \times \vec{u}) = 0, \quad \vec{\omega} = \nabla \times \vec{u}, \quad \nabla \cdot \vec{u} = 0 \quad \text{and} \quad \vec{u} \cdot \vec{n} = 0 \text{ on } \partial\Omega. \tag{2}$$

A time-splitting numerical scheme based on an alternating procedure between vorticity and velocity-Bernoulli pressure systems of equations will be constructed, similar to the approach followed in [6]. The conservation of energy, mass, vorticity and enstrophy will be proven and comparison with other numerical methods will be shown.

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