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VARIOUS VARIATIONAL FORMULATIONS FOR CURVE AND SURFACE INTERACTIONS

Alexander Konyukhov[1], **Karl Schweizerhof**[2]

[1] Institute of Mechanics, Karlsruhe Institute of Technology, Kaiserstrasse 12, Alexander.Konyukhov@kit.edu and http://www.ifm.kit.edu/14_203.php

[2] Institute of Mechanics, Karlsruhe Institute of Technology, Karl.Schweizerhof@kit.edu

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1 Kinematics of the Curve-To-Surface Interaction

Contact between the curve (geometrically representing either a rope, or a beam, or an edge of the solid) and the surface can be described in various ways: a) a direct approach via the Surface-To-Surface (STS) Mortar type algorithm; b) a special combination of Curve-To-Curve (CTC) and Segment-To-Analytical-Surface (STAS) Algorithm; c) a new variational equation based on consideration of contact in the Darboux basis.

1.1 Direct approach – STS Mortar type algorithm

The direct approach is just an implication of STS Mortar type algorithm in which a curve $\mathbf{r}_s(\eta)$ is assumed to be the "slave" part, as usually in any surface type contact algorithm. In this case all contact kinematic parameters are considered in the surface coordinate system $\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \mathbf{n}$ via convective coordinates ξ^2, ξ^2.

$$\mathbf{r}_s(\eta) = \boldsymbol{\rho}(\xi^1, \xi^2) + \xi^3 \mathbf{n}(\xi^1, \xi^2), \tag{1}$$

This approach, though, allows to capture the contact between a curve and a surface correctly, however, **does not allow to describe contact kinematics in details**, namely, to distinguish between the pulling and the dragging direction of the curve. In fact, this type can be regarded as "surrogate" type of modeling if a special algorithm is not available.

1.2 A special combination of Curve-To-Curve and Segment-To-Analytical-Surface Algorithm

This type of contact, in case of rigid surfaces, is the combination of the Surface-To-Analytical-Surface and the Curve-To-Curve contact kinematics, see in [1]. All kinematic parameters are formulated dually in the surface coordinate system eqn. (1) (via Gaussian coordinates ξ^1, ξ^2) and in the following Serret-Frenet curve coordinate system $\boldsymbol{\tau}, \boldsymbol{\nu}, \boldsymbol{\beta}$ (via coordinate η) similar to the Curve-To-Curve contact, see [2]:

$$\boldsymbol{\rho} = \mathbf{r}_s(\eta) + r\mathbf{e}, \quad \text{with } \mathbf{e} = \boldsymbol{\nu}(\eta)\cos\varphi + \boldsymbol{\beta}(\eta)). \tag{2}$$

Kinematical relationships during the contact can be obtained dually considering the relative velocity of the contact point during contact interaction in both coordinate systems eqns. (1, 2). After transformation we obtain the following components of the relative velocity vector: a) normal relative velocity during contact $\dot{r} = (\mathbf{v}_s - \mathbf{v}) \cdot \mathbf{n}$; b) pulling relative velocity $v_\tau = -(\boldsymbol{\rho}_i \cdot \boldsymbol{\tau})\dot{\xi}^i$ and c) dragging relative velocity $v_g = -(\boldsymbol{\rho}_i \cdot \mathbf{g})\dot{\xi}^i$. This approach allows to describe kinematics of the curve and surface interaction, such as pulling and dragging, precisely. For the finite element implementation both STS and CTC approaches are involved to obtain the corresponding residual as well as the corresponding tangent matrices, see in [1].

1.3 A new variational equation based on consideration of contact in the Darboux basis.

In this approach, a variational equation for the equilibrium of ropes on orthotropic rough surfaces is derived, using the consistent variational inclusion of frictional contact constraints via Karush-Kuhn-Tucker conditions expressed in Darboux basis. The weak forms is written as, see details in [3],

$$\int_0^l (T\delta\varepsilon + (\tilde{T} + q_\tau)\delta u_\tau + (k_g T + G + q_g)\delta u_g + (N + k_n T + q_n)\delta u_n) ds = 0, \tag{3}$$

where $\tilde{T}$ and G are corresponding pulling and dragging components, depending on the constitutive equation for the frictional force. The advantages of this approach are a) contact kinematics is precisely described; b) anisotropic friction law for the rope-surface interaction can be easily incorporated; c) new analytical solutions such as generalized Euler-Eytelwein formula, are obtained, see [3]. For the finite element implementation eqn. (3) is used, while Darboux transformation matrix should be used to transform the corresponding tangent matrices from both STS and CTC tangent matrices obtained earlier, see in [1].

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