

A STOCHASTIC CLOSURE APPROACH FOR LARGE EDDY SIMULATION

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In this work, we focus on a stochastic subgrid scale modelling strategy developed for application in finite volume Large Eddy Simulation (LES) codes. The approach is similar in spirit to earlier propositions, e.g., [4], but differs in terms of both the stochastic modelling ansatz and in terms of the underlying discontinuous Galerkin numerical techniques, e.g., [1].

Our concept is based on the integral conservation laws for mass, momentum and energy of a flow field that are universally valid for arbitrary control volumes. Thus, if the associated fluxes across its bounding surfaces are determined exactly, the equations capture the underlying physics of conservation correctly and guarantee an accurate prediction of the time evolution of the integral mean values.

We model the space-time structure of the fluxes to create a discrete formulation whose justification is independent of the underlying grid resolution. There are many alternatives for realizing this concept. Here, we directly aim at flux correction terms that should correct for the influence of the non-resolved small scale information. The classical closure problem would thus be reduced to the faithful reconstruction of spatio-temporal fluctuations of the fluxes across grid cell interfaces.

Advanced methods of time series analysis for the data-based construction of stochastic models with inherently non-stationary statistical properties are used to construct stochastic surrogate models for the non-resolved fluctuations from specific time series (cf. [2], [3]), leading to mixed deterministic-stochastic model formulation. Vector-valued auto-

regressive models with external influences (VARX-models) form the basis for the modeling approach. We realize non-stationary statistical properties of these models by allowing for time dependent switches between different fluctuation regimes which are represented by different, but fixed, sets of the stochastic model parameters. The LES-grid-averaged conserved quantities in the immediate vicinity of a given LES grid cell interface are interpreted as external influences in constructing the VARX surrogate model. In this fashion the stochastic models incorporate the information available from a typical numerical discretization stencil as would be used, e.g., in formulating a classical Smagorinsky closure.

The reconstruction capabilities of our modeling approach are tested against 3D turbulent channel flow data computed by direct numerical simulation (DNS) for an incompressible, isothermal fluid at Reynolds number $Re_\tau = 590$ (computed by [5]). Our approach particularly allows for the analysis of non-stationary and non-homogeneous data, resp., as the turbulent channel flow data are. Here, therefore, stationary/homogenous patterns, e.g. first order (Mean) and second order (Variance) statistics, often used in data analysis representations, could lead to biased results, as those moments typically do not represent the characteristics of inhomogeneous/instationary data.

Here, we present the outcome of our reconstruction test, and we show specifically results of the non-trivial time series data analysis. We found, surprisingly, that the deterministic model part alone is good enough to fit the flux correction terms well. That encourages us for the ambitious attempt at dynamic LES closure along these lines.

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