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A SHOCK ALIGNED CELL CENTERED GODUNOV SCHEME FOR EULERIAN HYDRODYNAMICS

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Classical Eulerian cell-centered Godunov schemes (see [5] and [8]),solve one-dimensional Riemann problems (RP) at the computational zone faces. The RP solutions provide the face and time step centered values of the variables required for the integration of the conservation laws. These RP are solved in the normal to the face direction. Such a procedure may introduce some mesh dependence in the solution. This defficiency of the Eulerian schemes is well known. See [7] on the use of a rotated Riemann solver to handle this issue. Here we propose a simpler solution. In most Lagrangian and ALE schemes the velocity is defined at the nodes and the momentum conservation integrated over a staggered zone. The new Staggered Mesh Godunov-SMG scheme [1, 2, 3, 4] solves the impact RP at the staggered zone faces **along the normal to the shock direction**. This direction is approximated to lie along the velocity difference across the face. The resulting SMG scheme better preserves the symmetry near shocks and it also naturally controls and prevents hourglass type of alternating instabilities. The aim of the present investigation is to apply similar ideas to the classical cell centered Eulerian schemes. In these schemes all variables including the velocities are zone centered. The data for the RP on the two sides of the zone faces are obtained from the corresponding zone centered values, using their slope limited gradient. Let $\hat{n}$ be the normal to the face. Now we assume, that if a shock is present at the face, it is directed along the velocity difference between the two neighbor zone centers $\hat{s} = \vec{\Delta} u_{12}/abs(\vec{\Delta} u_{12})$. Thus the RP is solved along this direction. The RP solution of pressure and specific internal energy, p^*, e^* are scalars, thus their value can be used directly while integrating the momentum and energy over the zone face. The velocity $\vec{u}^*$ is a vector which is directed along the shock direction $\hat{s}$. In the tangential direction $\hat{t}$, there is no shock, so that the velocity at the face can be obtained by interpolation of the velocity data at the face two sides. From this we can obtain the normal to face velocity to be used in the flux calculations as:

$$u_n = u^s(\hat{n} \cdot \hat{s}) + u^t(\hat{n} \cdot \hat{t})$$

We present several test problems to check the new method.

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