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LAMINATE ELEMENT METHOD FOR ELASTIC GUIDED WAVE DIFFRACTION SIMULATION

Evgeny V. Glushkov, Natalia V. Glushkova and Artem A. Eremin*

Institute for Mathematics, Mechanics and Informatics, Kuban State University
Stavropolskaya 149, Krasnodar, 350040, Russia, evg@math.kubsu.ru
http://www.immi.kubsu.ru

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The laminate element method (LEM) [1] is a special case of the method of fundamental solutions (MFS). In contrast to the conventional MFS, which assumes meshless expansion in terms of classical fundamental solutions for a homogeneous elastic space, the LEM is based on the use of the fundamental solutions for the elastic layered structure under consideration as a whole. Such basis functions called laminate elements (LEs) identically satisfy the governing equations in the sub-layers and the homogeneous boundary conditions at all plane-parallel surfaces and interfaces. Therefore, only conditions on the remaining non-planar parts of the domain's boundary, e.g., at obstacle boundaries, surface irregularities or structural edges, are to be approximated by LEs. This property considerably reduces the number of basis functions and thereby the computational expenses. It makes this method well-suitable for guided wave propagation and diffraction problems, especially in lengthy and expanded structures with localized obstacles.

A key point of successful LEM implementation is the development of efficient algorithms for LE calculation. Two of them, based on fast and stable numerical schemes for Green's matrix calculation in multilayered isotropic or anisotropic waveguides [2], are to be presented and discussed in the talk. In the first algorithm, the LE is constructed as a composition of the conventional matrix of singular fundamental solutions for a homogeneous elastic space with the properties of the layer, in which the LE is located, and non-singular terms for the fields reflected from the interfaces and external sides. In the alternative scheme, an additional plane-parallel boundary passing through the LE center is introduced, increasing the number of layers by one. In this case, the LE is the wave field generated by a point stress jump at this fictitious boundary, which is explicitly represented in terms of inverse Fourier integrals. It allows applying well-developed Green's matrix calculation algorithms without any modification.

The LEM implementation to guided wave diffraction problems has been performed in several ways. In the context of MFS, the scattered wavefield is sought as a linear combination of LEs centered at certain distance from the boundary to be approximated. Thus, a treatment of singularities is not required. Unknown expansion coefficients are determined either by the collocation method or using the Petrov-Galerkin scheme. This approach is cost effective and easier to implement, however in some situations, e.g., with small voids or narrow holes, it may become numerically unstable. Moreover, no straightforward algorithms exists to provide optimal LE placement along the boundaries.

Another approach is based on the indirect boundary element method (BEM) with a LE kernel. The use of LEs as boundary elements provides higher accuracy and much better stability, but it is more time consuming, especially in 3D, due to a multifold integration required to calculate mutual influence between non-complanar boundary elements. These expenses have been essentially reduced using the original trick proposed for a 2D scattering by inclined cracks [3]. It is based on a specific rotation in the Fourier parameter space, which brings multifold path integrals to onefold ones. As a result, Galerkin projection scheme has been developed and implemented, which provides closed representations for scattered wave fields in terms of 1D path Fourier integrals. Far-field asymptotic expansions derived from these integrals proved to be a convenient tool for fast and stable numerical simulation of 2D and 3D guided wave diffraction by localized obstacles (flaws) in laminate composite structures.

This technique as well as other LEM peculiarities are to be discussed and illustrated by the application to structural health monitoring (SHM) problems. In particular, guided wave diffraction by plane and volumetric obstacles such as rigid inclusions, holes and surface irregularities has been investigated. Examples of FEM and experimental verification of the proposed approaches are to be presented as well.

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