

## Constitutive Models for Plain and Fibre Reinforced Concrete Based on Micromechanical Solutions

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### Abstract

A constitutive model for plain concrete which employs continuum micromechanics solutions is presented. The model is subsequently extended to describe the behaviour of fibre reinforced concrete as well.

The model for plain concrete assumes a two-phase elastic composite, derived from an Eshelby solution and the Mori-Tanaka homogenization method, which comprises a matrix phase representing the mortar and spherical inclusions representing the coarse aggregate particles. Additionally, circular microcracks with various orientations are distributed within the matrix phase. Microcrack initiation can occur in any direction in the interfacial transition zone between aggregate particles and cement matrix and is governed by an exterior-point Eshelby solution. In each direction a local rough crack contact model is employed to simulate normal and shear behaviour of rough microcrack surfaces. An advantage of the two-phase formulation is that it is able to predict the build-up of tensile stresses within the matrix phase under uniaxial compression stresses thus allowing the model to naturally simulate compressive splitting cracks. Moreover, the implementation of the microcrack initiation criterion into the constitutive model enables the use of realistic material properties in order to obtain a correct cross-cracking response. It is shown, based on numerical predictions of uniaxial, biaxial and triaxial behaviour that the model captures key characteristics of concrete behaviour.

The model can be readily expanded so that it simulates the behaviour of fibre reinforced concrete. The addition of randomly distributed short fibres in a concrete matrix can enhance the tensile strength and significantly increase the fracture toughness of the composite. The underlying failure mechanism is governed by fibre pull-out. As a crack is formed and starts to open, the fibres crossing it undergo debonding and start to pull out from the concrete matrix and in this process they apply closure tractions on the crack faces thus stabilising the crack. Via these crack bridging mechanisms, the fibres continue to transfer stresses between the two crack faces until they are completely pulled out. The influence of fibres is incorporated into the model at a crack-plane level in a local constitutive relationship via an equivalent damage parameter that characterises the crack bridging state of the fibres. The evolution of the equivalent damage parameter for fibres is based on the micromechanics based crack bridging model of Lin and Li 1997 [1]. The performance of the extended model is shown through a series of numerical predictions of uniaxial tensile and compressive behaviour for different fibre types and volume fractions.

### References

[1] Z. Lin, V.C. Li, Crack bridging in fiber reinforced cementitious composites with slip-hardening interfaces. *Journal of the Mechanics and Physics of Solids*, 45(5), 763-787 (1997)